GRA 6035 Mathematics

Problems for Lecture 9

Key problems

Problem 1.
Let $0 < p < 1$ be a probability, with $q = 1 - p$, and let $a, b > 0$ be parameters such that $ap - bq > 0$. We consider the function $f(x) = p\ln(1 + ax) + q\ln(1 - bx)$ and the unconstrained optimization problem $\max f(x)$.

a) Show that the optimization problem has a solution for each value of the parameters a, b, p.
b) Compute the solution when $a = 2$, $b = 1$, and $p = 0.40$. What is the maximal value of f in this case?
c) Use the envelope theorem to compute $df^*(p)/dp$, and use this to estimate the new maximum value of f when $p = 0.43$.

Problem 2.
We consider the constrained optimization problem $\max f(x, y, z) = 4x^3 - 2y^3 + z^3$ when $x^3 + y^3 + z^3 \leq 8$.

a) Find the best candidate point in this problem.
b) Explain why this point is **not** a maximum point.

Problem 3.
We consider the constrained optimization problem $\max f(x, y, z) = 2x^2 - 4y^2 - 2z^2$ when $x^4 + y^4 + z^4 \leq 16$.

a) Use the EVT to show that this problem has a maximum point.
b) Show that the NDCQ is satisfied at all admissible points.
c) Find the maximum point and maximum value of f.
d) Use the envelope theorem to estimate the new maximum value of f when we
i) change the constraint to $x^4 + y^4 + z^4 \leq 20$ ii) change the objective function to $f(x, y, z) = x^2 - 4y^2 - 2z^2$
iii) change the constraint to $x^4 + y^4 + z^4 \leq 20$ and the objective function to $f(x, y, z) = x^2 - 4y^2 - 2z^2$

Problems from the Digital Workbook

Exercise problems	9.1 - 9.5 (full solutions in the workbook)
Exam problems	9.9, 9.10, 9.11ac (full solutions in the workbook)

Problems from Differential Equations

Problems in the appendix	A.1 - A.10 (full solutions on the web page)
Revision	Revise integrals from the appendix (or notes from FORK 1003)

Answers to key problems

Problem 1.
b) $x^* = 0.10,\ f^* \cong 0.0097$ c) $df^*(p)/dp = 0.2877,\ f^*(0.43) \cong 0.0183$

Problem 2.
a) $(x, y, z; \lambda) = (2, 0, 0; 4)$ with $f(2, 0, 0) = 32$
b) $f \to \infty$ when $x = z = 0$ and $y \to -\infty$ and $(0, y, 0)$ is admissible when $y \leq -2$ (for example, $f(0, -3, 0) = 81 > 32$)

Problem 3.
c) $(x, y, z; \lambda) = (\pm 2, 0, 0; 1/4)$ with $f(\pm 2, 0, 0) = 8$
d) *i)* $f_{\max} \cong 9$ *ii)* $f_{\max} \cong 4$ *iii)* $f_{\max} \cong 5$