Plan:

1. Eigenvalues and eigenvectors
2. Diagonalization

Reading:

[ME] 23.1–23.4, 23.6–23.7, (23.9)

Monday: 17–20 in A1-040 – Plenary Session I

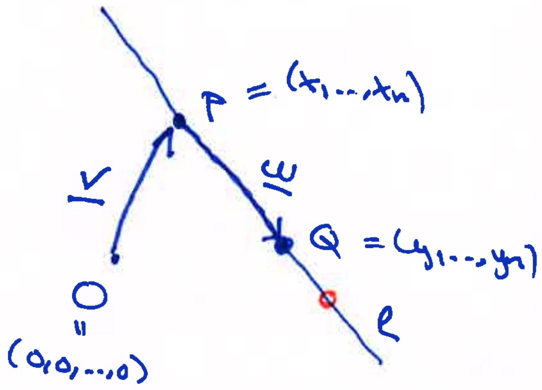
(no exercise session on Monday)

Review:

i) Parametric description of a line

$P = (x_1, \dots, x_n)$

$Q = (y_1, \dots, y_n)$



$$\underline{v} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad \underline{w} = \begin{pmatrix} y_1 - x_1 \\ y_2 - x_2 \\ \vdots \\ y_n - x_n \end{pmatrix}$$

Parametric description:

$$\underbrace{\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}}_{\underline{v}} + t \cdot \underbrace{\begin{pmatrix} y_1 - x_1 \\ \vdots \\ y_n - x_n \end{pmatrix}}_{\underline{w}} = \begin{pmatrix} x_1 + t(y_1 - x_1) \\ x_2 + t(y_2 - x_2) \\ \vdots \\ x_n + t(y_n - x_n) \end{pmatrix}$$

all numbers $t \in \mathbb{R}$ $\longleftrightarrow$ all pts on ℓ

$\underline{v}_1, \underline{v}_2, \dots, \underline{v}_n$ $\longleftrightarrow$ $A = \left(\underline{v}_1 \,|\, \underline{v}_2 \,|\, \dots \,|\, \underline{v}_n \right)$

m-vectors

ii) Linear independence

$\{\underline{v}_1, \dots, \underline{v}_n\}$ linearly independent if no vector is a linear combination of the others

$\Updownarrow$

$x_1 \cdot \underline{v}_1 + x_2 \underline{v}_2 + \dots + x_n \cdot \underline{v}_n = \underline{0}$ has only the trivial solution $\underline{x} = \underline{0}$.

$\Updownarrow$

$\text{rk } A = n$

$$\boxed{\underline{m=n}: \quad \text{rk } A = n \iff |A| \neq 0}$$

iii) Linear subspaces:

a) $V = \text{span}(\underline{v}_1, \dots, \underline{v}_n) = \text{Col}(A) \longleftarrow x_1 \underline{v}_1 + x_2 \underline{v}_2 + \dots + x_n \underline{v}_n$

$= (\underline{v}_1 | \dots | \underline{v}_n) \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = A \cdot \underline{x}$

b) $W = \text{Null}(A) = \{ \underline{x} : A\underline{x} = \underline{0} \} \longleftarrow$ all solutions of $A \cdot \underline{x} = \underline{0}$

Base of a linear subspace:

A base of V / W is a set of vectors that i) span V / W ii) linearly independent

Dimension:

The number of vectors in a base.

Formulas:

$$\boxed{\dim \underset{\substack{\uparrow \\ \text{Col}(A)}}{V} = \underline{\text{rk } A} \qquad \dim \underset{\substack{\uparrow \\ \text{Null}(A)}}{W} = \underline{n - \text{rk}(A)}}$$

Ex: Key problem Lecture 3

$\underline{v_1} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ $\underline{v_2} = \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}$ $\underline{v_3} = \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix}$ $\underline{v_4} = \begin{pmatrix} -1 \\ 3 \\ -7 \end{pmatrix}$ $\underline{v_5} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$

$\downarrow$

$$A = \begin{pmatrix} 1 & 1 & 1 & -1 & 1 \\ 2 & 4 & -2 & 3 & -1 \\ 1 & -1 & 5 & -7 & 3 \end{pmatrix} \longrightarrow \begin{array}{ccccc} v_1 & v_2 & & v_4 & \\ \end{array} \begin{pmatrix} 1 & 1 & 1 & -1 & 1 \\ 0 & 2 & -4 & 5 & -3 \\ 0 & 0 & 0 & -1 & -1 \end{pmatrix}$$

echelon form

a) $\underline{V = \text{span}(\underline{v_1}, \ldots, \underline{v_n}) = \text{Col}(A)}$:

$\{\underline{v_1}, \underline{v_2}, \underline{v_4}\}$ linearly independent

$$\left.\begin{aligned} \underline{v_3} &= 3\underline{v_1} - 2\underline{v_2} \\ \underline{v_5} &= 6\underline{v_1} - 4\underline{v_2} + \underline{v_4} \end{aligned}\right\} \text{lin. comb. of } \underline{v_1}, \underline{v_2}, \underline{v_4}$$

Concl: $\dim V = \underline{\underline{3}}$, base: $\{\underline{v_1}, \underline{v_2}, \underline{v_4}\}$

b) $\underline{W = \text{Null}(A)}$: $\dim W = \cancel{}\ n - \text{rk}\, A = \underline{\underline{2}}$

Solve: $A\underline{x} = \underline{0}$ (# free variables)

$$\begin{aligned} \underline{x_1} + x_2 + x_3 - x_4 + x_5 &= 0 \\ \underline{2x_2} - 4x_3 + 5x_4 - 3x_5 &= 0 \\ \underline{-x_4} - x_5 &= 0 \end{aligned} \quad \xrightarrow[\text{back subst.}]{} \quad \begin{aligned} x_4 &= -x_5 \\ x_2 &= 2x_3 + 4x_5 \\ x_1 &= -3x_3 - 6x_5 \end{aligned}$$

x_3, x_5 free

$$\underline{x} = \begin{pmatrix} -3x_3 - 6x_5 \\ 2x_3 + 4x_5 \\ x_3 \\ -x_5 \\ x_5 \end{pmatrix} = x_3 \cdot \underbrace{\begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}}_{= \underline{w_1}} + x_5 \cdot \underbrace{\begin{pmatrix} -6 \\ 4 \\ 0 \\ -1 \\ 1 \end{pmatrix}}_{= \underline{w_2}}$$

$\underline{x_3 = 1, x_5 = 0}$:

$-3\underline{v_1} + 2\underline{v_2} + \underline{v_3} = \underline{0}$

$\underline{v_3} = 3\underline{v_1} - 2\underline{v_2}$

$\underline{x_3 = 0, x_5 = 1}$:

$-6\underline{v_1} + 4\underline{v_2} \cancel{} + \underline{v_5} = 0$

$-\underline{v_4}$

$\underline{W = \text{Null}(A)}$: Base $\{\underline{w_1}, \underline{w_2}\}$ $\leftarrow$ automatically lin. indep. when x_3, x_5 are free

$= \text{span}(\underline{w_1}, \underline{w_2})$ $\dim W = \underline{\underline{2}}$

All linear subspaces are of the form span$(\underline{u}_1, \dots, \underline{u}_k)$ for some vectors $\underline{u}_1, \dots, \underline{u}_k$

Ex: $V = \text{span}(\underline{v}_1, \underline{v}_2, \underline{v}_4) \leftarrow \text{Col}(A)$

$W = \text{span}(\underline{w}_1, \underline{w}_2) \leftarrow \text{Null}(A)$

BREAK

① Eigenvectors and eigenvalues

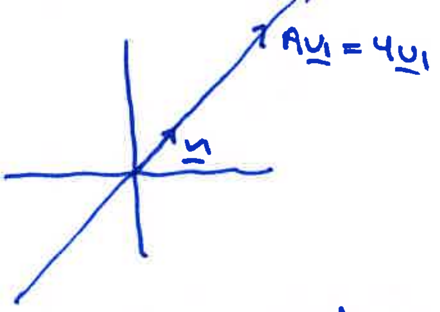
A $n \times n$ matrix

Defn: A number λ is called an eigenvalue of A if

$$\boxed{A \cdot \underline{u} = \lambda \underline{u}}$$

has non-trivial solutions $\underline{u} \neq \underline{0}$. The set $E_\lambda = \{\underline{u} : A\underline{u} = \lambda \underline{u}\}$ is called the eigenspace of A with eigenvalue λ, and all the vectors in E_λ are called eigenvectors for A with eigenvalue λ.

Ex: $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$



$$A \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix} \neq \lambda \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$A \cdot \underbrace{\begin{pmatrix} 1 \\ 1 \end{pmatrix}}_{\underline{u}} = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 4 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$(\lambda = 4)$

$$A \cdot \underline{u} = \lambda \underline{u}$$
$$A\underline{u} - \lambda \underline{u} = \underline{0}$$
$$(A - \lambda I)\underline{u} = \underline{0}$$

non-trivial solutions $\underline{u} \neq \underline{0}$

$\Updownarrow$

there are free var.

$\Updownarrow$

Characteristic equation: $\longrightarrow |A - \lambda I| = 0$

Method for finding eigenvalues:

λ eigenvalue for A if and only if $\underbrace{|A-\lambda I|=0}_{\text{characteristic eqn.}}$

$\boxed{\text{Eigenvalues for A: Solutions of } |A-\lambda I|=0.}$

Ex: $A=\begin{pmatrix}3&1\\1&3\end{pmatrix}$

$A-\lambda I=\begin{pmatrix}3&1\\1&3\end{pmatrix}-\lambda\cdot\begin{pmatrix}1&0\\0&1\end{pmatrix}=\begin{pmatrix}3-\lambda&1\\1&3-\lambda\end{pmatrix}$

$\begin{vmatrix}3-\lambda&1\\1&3-\lambda\end{vmatrix}=0$

$(3-\lambda)^2-1=0$

$9-6\lambda+\lambda^2-1=0 \rightarrow \lambda^2-6\lambda+8=0$

$\lambda=\frac{6\pm\sqrt{36-4\cdot 8}}{2}=\frac{6\pm 2}{2}$

Eigenvalues of A: $\lambda_1=4$ or $\lambda_2=2$

Ex: $A=\begin{pmatrix}a&b\\c&d\end{pmatrix}$

$\begin{vmatrix}a-\lambda&b\\c&d-\lambda\end{vmatrix}=0 \rightarrow (a-\lambda)(d-\lambda)-bc=0$

$ad-a\lambda-d\lambda+\lambda^2-bc=0$

$\lambda^2-\underbrace{(a+d)}_{\mathrm{tr}(A)}\lambda+\underbrace{(ad-bc)}_{|A| = \det(A)}=0$

$\mathrm{tr}(A)$ (trace, sum of diagonal entries)

$|A| = \det(A)$

$\boxed{\begin{array}{c}\text{Characteristic eqn for } 2\times 2 \text{ matrices}\\ \lambda^2-\mathrm{tr}(A)\cdot\lambda+\det(A)=0\end{array}}$

Ex: $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\lambda^2 - 0 \cdot \lambda + 1 = 0$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

no eigenvalues (among the real numbers)

Ex: $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$

$\begin{vmatrix} 3-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 3-\lambda \end{vmatrix} = 0$

$(2-\lambda) \cdot \begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 0$

$(2-\lambda) \cdot (\lambda^2 - 6\lambda + 8) = 0$

$-\lambda^3 + 8\lambda^2 - 20\lambda + 16 = 0$

$\lambda = 2$ or $\lambda^2 - 6\lambda + 8 = 0$

$\lambda = 4, \lambda = 2$

$\lambda_1 = \lambda_2 = 2, \lambda_3 = 4$

Summary: A $n \times n$ matrix

- the characteristic equation $|A - \lambda I| = 0$ is an n'th order polynomial equation
- the maximal number of eigenvalues is n (counted with multiplicity)
- if the char. eqn. has n eigenvalues $\lambda_1, \lambda_2, \dots, \lambda_n$ then
  $|A| = \lambda_1 \cdot \lambda_2 \cdot \ldots \cdot \lambda_n$
  $\mathrm{tr}(A) = \lambda_1 + \lambda_2 + \ldots + \lambda_n$

$n=2$: $\lambda^2 - \mathrm{tr}(A)\lambda + \det(A) = 0$

λ_1, λ_2: $\lambda_1 \cdot \lambda_2 = \det(A)$

$\lambda_1 + \lambda_2 = \mathrm{tr}(A)$

special case of

Method for finding eigenvectors

All eigenvectors for the eigenvalue λ:

$$E_\lambda = \text{Null}(A - \lambda I) \leftarrow \text{Solve } (A-\lambda I)\underline{v} = \underline{0}$$

Ex: $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

Eigenvalues: $\lambda_1 = 4, \quad \lambda_2 = 2$

$\lambda = 4$:

$A\underline{v} = 4\underline{v}$

$A\underline{v} - 4I\underline{v} = \underline{0}$

$(A - 4I)\cdot\underline{v} = \underline{0}$

$E_4 = \text{Null}(A - 4I)$

$\begin{pmatrix} 3-4 & 1 \\ 1 & 3-4 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

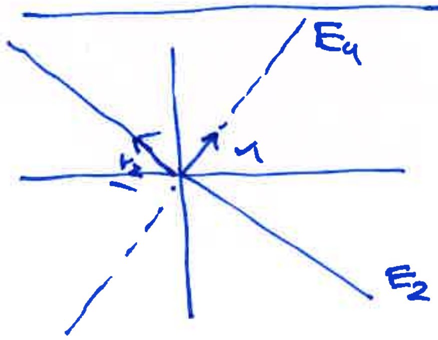
$\begin{pmatrix} -1 & 1 \\ \cancel{1} & \cancel{-1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$-x + y = 0$, y free

$x = y$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ y \end{pmatrix} = y \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ base



$\lambda = 2$: $E_2 = \text{Null}(A - 2I)$

$\begin{pmatrix} 3-2 & 1 \\ 1 & 3-2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 1 & 1 \\ \cancel{1} & \cancel{1} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$x + y = 0$, y free

$x = -y$

$v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ base $\quad \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -y \\ y \end{pmatrix} = y \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

Base of E_4: $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

Base of E_2: $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$

BREAK

Result: If λ is an eigenvalue of A with multiplicity m, then

$$1 \leq \underbrace{\dim E_\lambda}_{\substack{\#\text{ free var.} \\ \text{in } (A-\lambda I)\underline{v}=\underline{0}}} \leq m$$

$\underline{m=1}$: $\dim E_\lambda = 1$ — one free variable

$\underline{m=2}$: $\begin{cases} \dim E_\lambda = 1 & \text{(one free var.)} \\ \text{or} & \\ \dim E_\lambda = 2 & \text{(two free var.)} \end{cases}$

$\underline{\text{Ex}}$: $A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$

$$|A - \lambda I| = 0$$

$$(2-\lambda)\cdot(\lambda^2 - 6\lambda + 8) = 0$$

$$(2-\lambda)\cdot(\lambda-4)(\lambda-2) = 0$$

$$-\underbrace{(\lambda-2)(\lambda-2)}_{\substack{\lambda=2 \\ m=2}}\underbrace{(\lambda-4)}_{\substack{\lambda=4 \\ m=1}} = 0$$

$\underline{\lambda = 2}$: $E_2 = \text{Null}(A - 2I)$

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$x + z = 0$
y, z free

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ y \\ z \end{pmatrix} = y \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

two free var.
$\dim E_2 = 2$

(2) Diagonalization

A $n \times n$ matrix

<u>Defn:</u> A is <u>diagonalizable</u> if there is an invertible matrix P and a diagonal matrix D such that

$$P^{-1}AP = D$$

<u>Method:</u>

- Find all eigenvalues and eigenvectors of A
- Let $P = (\underline{v_1} | \underline{v_2} | \cdots | \underline{v_n})$ where $\{\underline{v_i}\}$ consists of base vectors for each E_λ.

$$D = \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

eigenvalues of A, such that $A \cdot \underline{v_1} = \lambda_1 \underline{v_1}$
$A \cdot \underline{v_2} = \lambda_2 \cdot \underline{v_2}$
$\vdots$

<u>This works if:</u> (conditions for diagonalizability)

- there are n eigenvalues, counted with multiplicity
- there are enough eigenvectors; this means that $\dim E_\lambda = m$ for each eigenvalue of multiplicity m.

<u>Why?</u> We can rewrite $P^{-1}AP = D$ as $\boxed{AP = PD}$

$$A \cdot (\underline{v_1} | \underline{v_2} | \cdots | \underline{v_n}) = (\underline{v_1} | \underline{v_2} | \cdots | \underline{v_n}) \cdot \begin{pmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_n \end{pmatrix}$$

$$(A\underline{v_1} | A\underline{v_2} | \cdots | A\underline{v_n}) = (\lambda_1\underline{v_1} | \lambda_2\underline{v_2} | \cdots | \lambda_n\underline{v_n})$$

$\underline{v_i}$ is eigenvector with eigenvalue λ

Ex: $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$\lambda^2 + 1 = 0$

no eigenvalues

A not diag. since it has too few eigenvalues

Ex: $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Eigenvalues:

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda_1 = \lambda_2 = 1 \quad (m=2)$

$\lambda = \dfrac{2 \pm \sqrt{4 - 4\cdot 1}}{2}$

$\lambda_1 = \lambda_2 = 1 \quad (m=2)$

$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

enough eigenvalues

$E_1 = \text{Null}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$:

$y = 0$, x free
$0 = 0$

$\dim E_1 = 1$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$P = \begin{pmatrix} 1 & ? \\ 0 & ? \end{pmatrix}$

not enough eigenvectors

A not diagonalizable

Summary:

A $n \times n$ matrix

A is diagonalizable

$\Updownarrow$

i) There are n eigenvalues $\lambda_1, \dots, \lambda_n$ counted with multiplicity.

ii) For each eigenvalue λ of mult. $m > 1$, $\dim E_\lambda = m$.

Theorem:

If A is symmetric, then it is diagonalizable.

Remember: A symmetric means $A^T = A$

Ex: $A = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$ A is diagonalizable

Ex: Is $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ diagonalizable?

Short answer: Yes, since it is symmetric

Eigenvalues:

$$|A - \lambda I| = \begin{vmatrix} 4-\lambda & 0 & 1 \\ 0 & 3-\lambda & 0 \\ 1 & 0 & 4-\lambda \end{vmatrix} = (3-\lambda)\begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = (3-\lambda)(\lambda^2 - 8\lambda + 15) = 0$$

$\lambda = 3$ or $\lambda^2 - 8\lambda + 15 = 0$

$\lambda = 3, \lambda = 5$

$$D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

($\lambda = 3$ has multipl. $m = 2$)

Eigenvectors:

$\lambda = 3$: $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ \cancel{1} & \cancel{0} & \cancel{1} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$x + z = 0$

y, z free

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -z \\ y \\ z \end{pmatrix} = y\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + z\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

Base: $v_1 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ $v_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$

$\lambda = 5$: $\begin{pmatrix} -1 & 0 & 1 \\ 0 & -2 & 0 \\ \cancel{1} & \cancel{0} & \cancel{-1} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-x + z = 0$

$-2y = 0$

z free

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ 0 \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Base: $v_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

Bases: E_3: $\left\{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}\right\}$

E_5: $\left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}\right\}$

$$P = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

Conclusion: A is diagonalizable

$P^{-1}AP = D$ with $P = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ $D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}$