Key Problems

Problem 1.
Solve the exact differential equations:

a) $3t^2 - 2t + 2y \cdot y' = 0$ b) $2y - 3t^2 + 2(y+t)y' = 0$ c) $\dfrac{y(1-2\ln t)}{t^3} + \dfrac{\ln t}{t^2} \cdot y' = 0$

Problem 2.
Find the equilibrium states and determine their stability. Sketch the solution curve $y = y(t)$.

a) $y' = 6 - 2y$ b) $y' = y^2 - 4$ c) $y' = 5y(1 - y/10)$

Problem 3.
Solve the differential equations:

a) $y'' + 6y' - 16y = 16t - 22$ b) $y'' + 6y' + 9y = 4e^{-t}$

c) $y'' - 3y' + 2y = 3e^{2t}$ d) $y'' - y = t^2$

Problems from the Workbook and Differential Equations

Workbook [W]	10.13 - 10.16, 11.1 - 11.17 (full solutions in the workbook)
Differential equations [DE]	1.20 - 1.34 (full solutions on the web page)

Answers to Key Problems

Problem 1.

a) $y = \pm\sqrt{t^2 - t^3 + C}$ b) $y = -t \pm \sqrt{t^2 + t^3 + C}$ c) $y = \dfrac{Ct^2}{\ln t}$

Problem 2.

a) $y_e = 3$ is globally asymptotically stable

b) $y_e = -2$ is stable (but not globally asymptotically stable), $y_e = 2$ is unstable

c) $y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)

Problem 3.

a) $y = C_1\, e^{-8t} + C_2\, e^{2t} + 1 - t$ b) $y = C_1\, e^{-3t} + C_2 t e^{-3t} + e^{-t}$

c) $y = C_1\, e^{2t} + C_2 e^{t} + 3te^{2t}$ d) $y = C_1\, e^{t} + C_2\, e^{-t} - t^2 - 2$