Key Problems

Problem 1.
Let $0 < p < 1$ be a probability, let $q = 1 - p$, and let $a,b > 0$ be parameters such that $ap - bq > 0$. We consider the unconstrained optimization problem $\max f(x) = p \ln(1 + ax) + q \ln(1 - bx)$ with parameters p,a,b.

a) Show that the optimization problem has a solution.
b) Compute the solution when $a = 2$, $b = 1$, and $p = 0.40$. What is the maximum value of f in this case?
c) Use the envelope theorem to compute $df^*(p)/dp$, and estimate the new maximum value of f when $p = 0.43$.

Problem 2.
We consider the constrained optimization problem $\max f(x,y,z) = 4x^3 - 2y^3 + z^3$ when $x^3 + y^3 + z^3 \leq 8$.

a) Find the best candidate point in this problem.
b) Explain why this point is **not** a maximum point.

Problem 3.
We consider the constrained optimization problem $\max f(x,y,z) = 2x^2 - 4y^2 - 2z^2$ when $x^4 + y^4 + z^4 \leq 16$.

a) Find the maximum point and maximum value of f.
b) Use the envelope theorem to estimate the new maximum value of f when we
i) change the constraint to $x^4 + y^4 + z^4 \leq 20$ ii) change the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$
iii) change the constraint to $x^4 + y^4 + z^4 \leq 20$ and the objective function to $f(x,y,z) = x^2 - 4y^2 - 2z^2$

Problems from the Workbook and Differential Equations

Workbook [W]	9.1 - 9.5, 9.9 - 9.10, 9.11ac (full solutions in the workbook)
	9.12 - 9.14 (difficult problems for those interested)
Differential equations [DE]	Revise integrals from Appendix A in [DE]
	Problems A.1 - A.10 in [DE] (full solutions on the web page)

Answers to Key Problems

Problem 1.
a) f is concave and $x = \dfrac{ap - bq}{ab}$ is stationary
b) $x^* = 0.10,\ f^* \cong 0.0097$
c) $df^*(p)/dp = 0.2877,\ f^*(0.43) \cong 0.0183$

Problem 2.
a) $(x,y,z;\lambda) = (2,0,0;4)$ with $f(2,0,0) = 32$
b) We have that $(0,y,0)$ is admissible when $y \leq -2$, and $f \to \infty$ when $x = z = 0$ and $y \to -\infty$.

Problem 3.
a) $(x,y,z;\lambda) = (\pm 2,0,0;1/4)$ with $f(\pm 2,0,0) = 8$
b) *i)* $f_{\max} \cong 9$ *ii)* $f_{\max} \cong 4$ *iii)* $f_{\max} \cong 5$