	Plan	
1	Key Problems:	1.1c, 1.3, 2.1, 2.3, 2.4, 3.3, 3.4bc, 4.1ef, 4.3, 4.4c
2	Exam Problems:	Midterm 10/2019 Q8, 01/2020 Q8
3	Textbook Problems:	3.11, 3.12b, 3.13, 3.14, 4.6

1 Key Problems

1.1c:

$$\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 4 & 8 \\ 1 & 3 & 1 & 5 & 18 & 28 \\ 2 & 4 & 2 & 9 & 31 & 48 \end{array}\right) \begin{matrix} -1 \\ -2 \end{matrix}$$

$$\to \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 4 & 8 \\ 0 & 2 & 0 & 4 & 14 & 20 \\ 0 & 2 & 0 & 7 & 23 & 32 \end{array}\right) -1$$

$$\to \left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 4 & 8 \\ 0 & 2 & 0 & 4 & 14 & 20 \\ 0 & 0 & 0 & 3 & 9 & 12 \end{array}\right)$$

$x_1 + x_2 + x_3 + x_4 + 4x_5 = 8$

$2x_2 + 4x_4 + 14x_5 = 2$

$3x_4 + 9x_5 = 1$

$x_3 = s \quad x_5 = t$

free variables

$$\frac{3x_4}{3} = \frac{12 - 9x_5}{3} \qquad x_4 = 4 - 3x_5$$

$$x_4 = 4 - 3t$$

$$\frac{2x_2}{2} = \frac{20 - 4(4-3t) - 14t}{2} = \frac{4 - 2t}{2} \qquad x_2 = 2 - t$$

$$x_1 = 8 - (2-t) - s - (4-3t) - 4t = 2 - s$$

$$\underline{x} = (x_1, x_2, x_3, x_4, x_5) = (2-s, 2-t, s, 4-3t, t)$$

1.3.

$$\begin{aligned} x+y+2z+4w &= 6 \\ x+2y+4z-2w &= 9 \\ x+3y+9z+7w &= 24 \\ x-y-z+w &= 0 \end{aligned} \longleftarrow \boxed{x+w = y+z}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 6 \\ 1 & 2 & 4 & -2 & 9 \\ 1 & 3 & 9 & 7 & 24 \\ 1 & -1 & -1 & 1 & 0 \end{array}\right) \begin{array}{l} -1 \\ -1 \\ -1 \end{array} \to \left(\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 6 \\ 0 & 1 & 2 & -6 & 3 \\ 0 & 2 & 7 & 3 & 18 \\ 0 & -2 & -3 & -3 & -6 \end{array}\right) \begin{array}{l} -2 \\ 2 \end{array}$$

$$\to \left(\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 6 \\ 0 & 1 & 2 & -6 & 3 \\ 0 & 0 & 3 & 15 & 12 \\ 0 & 0 & 1 & -15 & 0 \end{array}\right) -1/3 \to \left(\begin{array}{cccc|c} 1 & 1 & 2 & 4 & 6 \\ 0 & 1 & 2 & -6 & 3 \\ 0 & 0 & 3 & 15 & 12 \\ 0 & 0 & 0 & -20 & -4 \end{array}\right)$$

$-20w = -4 \qquad w = 1/5$

$z = 4 - 5\cdot w = 4 - 5(1/5) = 3$

$y = 3 - 2\cdot 3 + 6\cdot 1/5 = 6/5 - 15/5 = -9/5$

$x = \cancel{6} - (-9/5) \cancel{-2\cdot 3} - 4\cdot 1/5 = 1$

$(x,y,z,w) = (1, -9/5, 3, 1/5)$

Alt method:

w free

$z = 4 - 5w$

$y = 3 - 2(4-5w) + 6w = -5 + 16w$

$x = 6 - (-5+16w) - 2(4-5w) - 4w$

$= 3 - 10w$

$x + w = y + z$

$3 - 9w = -1 + 11w$

$\frac{-20w}{-20} = \frac{-4}{-20}$

$w = 1/5$

2.1. $A = (\underline{v}_1 | \underline{v}_2 | \underline{v}_3 | \underline{v}_4 | \underline{v}_5) = \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 3 & 3 & 3 & 4 & 2 \\ 4 & 4 & 4 & 5 & 3 \end{pmatrix}$ -3, -4

$\rightarrow \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix}$ -1 $\rightarrow \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & -2 & 4 & 5 & 3 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix}$ 4

$\rightarrow \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & -2 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$

a) $\{\underline{v}_1, \underline{v}_2\}$: Linearly independent since all col's have pivots

Span$(\underline{v}_1, \underline{v}_2)$: $\{\underline{v}_1, \underline{v}_2\}$ is base, dim = 2

b) $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$: Not lin. independent ($\underline{v}_3$ is lin.comb. of $\underline{v}_1, \underline{v}_2$

Span$(\underline{v}_1, \underline{v}_2, \underline{v}_3)$: Base $\{\underline{v}_1, \underline{v}_2\}$, dim = 2

c) $\{\underline{v}_2, \underline{v}_3, \underline{v}_4\}$: $\begin{pmatrix} -1 & 5 & 6 \\ -2 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}$ -2 $\rightarrow \begin{pmatrix} -1 & 5 & 6 \\ 0 & -6 & -7 \\ 0 & 0 & 1 \end{pmatrix}$

$\{\underline{v}_2, \underline{v}_3, \underline{v}_4\}$ lin. independent, base, dim = 3

2.3

$$A = \begin{array}{c} \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 2 & 4 & -2 & -2 & -2 \\ 3 & 5 & -1 & -1 & -1 \end{pmatrix} \begin{matrix} \\ \big]{-2} \\ \big]{-3} \end{matrix} \\ \quad \underline{v_1} \quad \underline{v_2} \quad \underline{v_3} \quad \underline{v_4} \quad \underline{v_5} \end{array}$$

b) $\underline{V = \mathrm{Col}(A)}$:

$$\begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & 6 & -12 & -14 & -10 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix} \rfloor -1$$

$\dim \mathrm{Col}(A) = \underline{\underline{3}} = \mathrm{rk}(A)$

$\downarrow$

$\underline{\text{Base}}$: $\{\underline{v_1}, \underline{v_2}, \underline{v_4}\}$.

$V = \mathbb{R}^3$ $\underline{\text{3-dim space}}$ in $\mathbb{R}^3$

$$\begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & -2 & 4 & 5 & 3 \\ 0 & 8 & -16 & -19 & -13 \end{pmatrix} \big]{4}$$

a) $\underline{\mathrm{Null}(A)}$: $\boxed{\text{Sol. of } A \cdot \underline{x} = \underline{0}}$

$\downarrow$

$\dim \mathrm{Null}(A) = 2 = n - \mathrm{rk}(A)$

$$\begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 0 & -2 & 4 & 5 & 3 \\ 0 & 0 & 0 & 1 & -1 \end{pmatrix}$$

$\underline{\text{Base}}$: $\underline{w_1} = (-3, 2, 1, 0, 0)$

$\{\underline{w_1}, \underline{w_2}\}$ $\underline{w_2} = (-6, 4, 0, 1, 1)$

$$\underline{x_1} - x_2 + 5x_3 + 6x_4 + 4x_5 = 0$$
$$-2\underline{x_2} + 4x_3 + 5x_4 + 3x_5 = 0$$
$$\underline{x_4} - x_5 = 0$$

$\left.\begin{matrix} x_3 = s \\ x_5 = t \end{matrix}\right\}$ free

$V \subseteq \mathbb{R}^5$ $\underline{\text{a plane}}$

$x_4 = x_5 = \underline{t}$

$$\frac{-2x_2}{-2} = -4s - 5t - 3t = \frac{-4s - 8t}{-2} \qquad x_2 = \underline{2s + 4t}$$

$$x_1 = (2s + 4t) - 5s - 6(t) - 4t = \underline{-3s - 6t}$$

$$\underline{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -3s - 6t \\ 2s + 4t \\ s \\ t \\ t \end{pmatrix} = s \cdot \underline{\begin{pmatrix} -3 \\ 2 \\ 1 \\ 0 \\ 0 \end{pmatrix}} + t \cdot \begin{pmatrix} -6 \\ 4 \\ 0 \\ 1 \\ 1 \end{pmatrix}$$

24: A 8×8 matrix, $rk(A)=7$

a) $\dim \text{Null}(A) = n - rk(A) = 8-7 = \underline{1}$

b) $\dim \text{Col}(A) = rk(A) = \underline{7}$

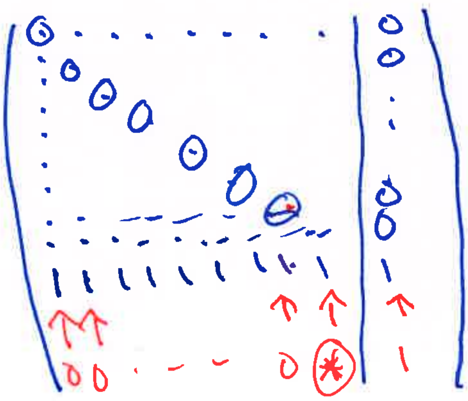
d) # Sol's of $A\underline{x}=\underline{0}$: inf. many (one degree of freedom)

e) # Sol's of $A\underline{x}=\underline{b}$:

$* = 0$: one free variable, inf. many solutions $\Longleftrightarrow$ if $\underline{b}$ is in Col(A)

$* \neq 0$: no solutions

f) #Sol's of $A\underline{x}=\underline{0}$ s.t. $x_1+x_2+\dots+x_8=1$



$(*) = 0$: no solutions

$(*) \neq 0$: one solution

3.3

a) $A = \begin{pmatrix} 4 & 1 & 1 & 3 & 7 \\ 2 & 1 & 0 & 1 & 0 \\ 1 & 0 & 3 & 1 & 0 \end{pmatrix}$

$\underline{v}_1 \quad \underline{v}_2 \quad \underline{v}_3 \quad \underline{v}_4 \quad \underline{v}_5$

$M_{123,123} = 1(-1) + 3\cdot 2 = 5 \neq 0$

$\Rightarrow rk(A) = \underline{\underline{3}}$

Base Col(A): $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$

c) $A = \begin{pmatrix} 1 & 4 & -3 & 1 \\ 2 & 7 & 1 & 2 \\ 1 & 3 & 4 & 1 \end{pmatrix}$

$\underline{v}_1 \quad \underline{v}_2 \quad \underline{v}_3 \quad \underline{v}_4$

$M_{123,123} = 1\cdot 25 - 4\cdot 7 - 3\cdot(-1)$
$= 0$

$R(3) = R(2) - R(1) \Rightarrow$ All 3-minors are zero
$\Rightarrow rk(A) < 3$

$M_{12,12} = 7 - 8 = -1 \neq 0 \Rightarrow rk(A) = \underline{\underline{2}}$

Base Col(A): $\{\underline{v}_1, \underline{v}_2\}$

3.4

b) $A = \begin{pmatrix} 1 & 3 & 2 & -1 \\ s & 3 & 0 & 1 \\ 4 & 6 & 2 & 0 \end{pmatrix}$

$M_{123,123} = -s(-6) + 3\cdot(-6)$
$= 6s - 18 = \underline{6(s-3)}$

$s=3$: $M_{123,123} = 0$: $rk A = 2$

$s \neq 3$: $M_{123,123} \neq 0$: $rk A = \underline{\underline{3}}$

$M_{123,234} = -3\cdot 2 - 1\cdot(6-12) = 0$

$\begin{pmatrix} 1 & 3 & 2 & -1 \\ 3 & 3 & 0 & 1 \\ 4 & 6 & 2 & 0 \end{pmatrix}$ $R(3) = R(1) + R(2) \Rightarrow$ all 3-minors $= 0$ $\Rightarrow rk A < 3$

$M_{12,12} = 3 - 9 \neq 0 \Rightarrow rk A = 2$ when $s = 3$

c) $A = \begin{pmatrix} 1 & a & b \\ a & b & c \end{pmatrix}$: $\text{rk} A < 2 \Leftrightarrow$ all 2-minors are zero

$M_{12,12} = b - a^2 = 0 \Rightarrow \underline{b = a^2}$

$M_{12,23} = ac - b^2 = 0 \Rightarrow a \cdot a^3 - (a^2)^2 = 0$ ok

$M_{12,13} = c - ab = 0 \Rightarrow c = ab = a \cdot a^2 = a^3$

$\underline{c = a^3}$

$\Downarrow$

$\text{rk} A < 2$ when $b = a^2$ and $c = a^3$, $\Rightarrow \text{rk} A = 1$

$\text{rk} A = 2$ otherwise

$$\text{rk} A = \begin{cases} 1, & b = a^2 \text{ and } c = a^3 \\ 2, & \text{otherwise} \end{cases}$$

4.1 e) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

$-\lambda^3 + 0 \cdot \lambda^2 - (-3)\lambda + 2 = 0$

$\underline{-\lambda^3 + 3\lambda + 2 = 0}$

$|A| = -1(-1) + 1 \cdot 1 = 2$

$\lambda = -1$ is a solution

$(\lambda+1)(-\lambda^2 + \lambda + 2) = 0$

$\underline{\lambda = -1}$ or $\lambda = \dfrac{-1 \pm \sqrt{1+8}}{-2}$

$= \dfrac{-1 \pm 3}{-2}$

$\lambda = \underline{-1}$, $\lambda = \underline{2}$

Alt: $\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$-\lambda(\lambda^2 - 1) - 1(-\lambda - 1) + 1 \cdot (1 + \lambda) = 0$

$-\lambda(\lambda+1)(\lambda-1) + (\lambda+1) + (\lambda+1) = 0$

$(\lambda+1) \cdot [-\lambda(\lambda-1) + 2] = 0$

$(\lambda+1)(-\lambda^2 + \lambda + 2) = 0$

$\underline{\lambda = -1}$, $\underline{\lambda = -1}$, $\underline{\lambda = 2}$

Eigenvalues: $\lambda = -1$ (mult. 2), $\lambda = 2$ (mult 1)

Eigenvectors:

$\lambda = -1$: $A - \lambda I = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \to \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad x + y + z = 0$, y, z free $\Rightarrow x = -y - z$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -y-z \\ y \\ z \end{pmatrix} = y \cdot \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + z \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$$

$\lambda = 2$: $A - \lambda I = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \end{pmatrix}$

$$\to \begin{pmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{pmatrix} \to \begin{pmatrix} 1 & -2 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{pmatrix}$$

$x = z$, $y = z$, z free

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ z \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$D = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad P = \begin{pmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

f) $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$

A upper triangular $\Rightarrow$ Eigenvalues = diagonal entries

$\begin{vmatrix} -\lambda & 1 & 1 \\ 0 & -\lambda & 1 \\ 0 & 0 & -\lambda \end{vmatrix} = 0 \qquad (-\lambda)^3 = 0 \qquad -\lambda^3 = 0$

$\lambda_1 = \lambda_2 = \lambda_3 = 0$

(mult 3)

E_0: $\begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ x free, $y = 0$, $z = 0$

$\Downarrow$

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

A is diagonalizable since it is symmetric

3\. Eigenvalues:

$\begin{vmatrix} 1-\lambda & 0 & 0 & 4 \\ 0 & 2-\lambda & 3 & 0 \\ 0 & 3 & 2-\lambda & 0 \\ 4 & 0 & 0 & 1-\lambda \end{vmatrix} = 0$

$(1-\lambda) \cdot \begin{vmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} - 4 \cdot \begin{vmatrix} 0 & 2-\lambda & 3 \\ 0 & 3 & 2-\lambda \\ 4 & 0 & 0 \end{vmatrix} = 0$

$(1-\lambda)(1-\lambda) \cdot \begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} - 4 \cdot 4 \cdot \begin{vmatrix} 2-\lambda & 3 \\ 3 & 2-\lambda \end{vmatrix} = 0$

$\left((1-\lambda)^2 - 16\right) \cdot \left((2-\lambda)^2 - 9\right) = 0$

$\lambda^2 - 2\lambda - 15 = 0$ or $\lambda^2 - 4\lambda - 5 = 0$

$(\lambda - 5)(\lambda + 3) = 0 \qquad (\lambda - 5)(\lambda + 1) = 0$

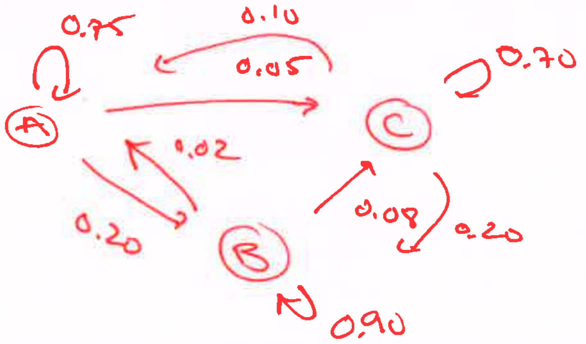
$\lambda_1 = \lambda_2 = 5, \ \lambda_3 = -1, \ \lambda_4 = -3$

4.4c

$$A = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{ccc} A & B & C \\ \end{array}$$

$$A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

Each column has sum = 1 $\Rightarrow \lambda = +1$ is an eigenvalue



$\lambda=1$:

$$\begin{pmatrix} -0.25 & 0.02 & 0.10 \\ 0.20 & -0.10 & 0.20 \\ 0.05 & 0.08 & -0.30 \end{pmatrix} \to \begin{pmatrix} 0.05 & 0.08 & -0.30 \\ 0.20 & -0.10 & 0.20 \\ -0.25 & 0.02 & 0.10 \end{pmatrix} \begin{matrix} -4 \\ \\ 5 \end{matrix}$$

$$\to \begin{pmatrix} 0.05 & 0.08 & -0.30 \\ 0 & -0.42 & 1.40 \\ 0 & 0.42 & -1.40 \end{pmatrix} \begin{matrix} \\ 1 \\ \end{matrix} \to \begin{pmatrix} 0.05 & 0.08 & -0.30 \\ 0 & -0.42 & 1.40 \\ 0 & 0 & 0 \end{pmatrix}$$

z free $\quad -0.42y + 1.40z = 0$

$$y = \frac{1.40z}{0.42} = \frac{140}{42}z = \frac{10}{3}z$$

free variables confirm that $\lambda=1$ is eigenvalue!

$$0.05x + 0.08\left(\frac{10}{3}z\right) - 0.30z = 0$$

$$0.05x = 0.30z - \frac{0.8}{3}z \qquad 1.20$$

$$x = 6z - \frac{16}{3}z = \frac{2}{3}z$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{2}{3}z \\ \frac{10}{3}z \\ z \end{pmatrix} = z \cdot \begin{pmatrix} 2/3 \\ 10/3 \\ 1 \end{pmatrix} \qquad v_1 = \begin{pmatrix} 2 \\ 10 \\ 3 \end{pmatrix}$$

$(z=3)$

$\lambda_1 = 1$, λ_2, λ_3 unknown:

$$1 \cdot \lambda_2 \cdot \lambda_3 = \det(A) = 0.455$$
$$1 + \lambda_2 + \lambda_3 = \operatorname{tr}(A) = 2.35$$

compute!

$$\Downarrow$$

$$\lambda_2 \cdot \lambda_3 = 0.455$$
$$\lambda_2 + \lambda_3 = 1.35$$

$$\Downarrow$$

$$\lambda^2 - 1.35\lambda + 0.455 = 0$$

quadratic formula

$$\underline{\lambda_2 = 0.70}, \quad \underline{\lambda_3 = 0.65}$$

Gauss:

$E_{0.70}$: $z \cdot \begin{pmatrix} -8/3 \\ 5/3 \\ 1 \end{pmatrix}$, $\quad v_2 = \begin{pmatrix} -8 \\ 5 \\ 3 \end{pmatrix}$ $(z = 3)$

$E_{0.65}$: $z \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, $\quad v_3 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $(z = 1)$

$$D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.70 & 0 \\ 0 & 0 & 0.65 \end{pmatrix}$$

$$P = \begin{pmatrix} 2 & -8 & -1 \\ 10 & 5 & 0 \\ 3 & 3 & 1 \end{pmatrix}$$

$$P^{-1} = \frac{1}{|P|} \operatorname{adj}(P)$$

$$P^{-1} = \frac{1}{15} \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -2 \\ 3 & -6 & 18 \end{pmatrix}$$

$$A^n = (PDP^{-1})^n = P \cdot D^n P^{-1} \to \begin{pmatrix} 2 & -8 & -1 \\ 10 & 5 & 0 \\ 3 & 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{15} \begin{pmatrix} 1 & 1 & 1 \\ -2 & 1 & -2 \\ 3 & -6 & 18 \end{pmatrix}$$

$$= \frac{1}{15} \begin{pmatrix} 2 & 2 & 2 \\ 10 & 10 & 10 \\ 3 & 3 & 3 \end{pmatrix} = \left(\underline{v} \,|\, \underline{v} \,|\, \underline{v} \right) \quad \text{with } \underline{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$$

(equilibrium state)

$$A^n \cdot \underline{v_0} \to \underline{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$$

Midterm 01/2020, Q8:

$\underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \quad \underline{v}_2 = \begin{pmatrix} 7 \\ 0 \\ 1 \\ 4 \end{pmatrix} \quad \underline{v}_3 = \begin{pmatrix} 4 \\ -3 \\ 1 \\ 1 \end{pmatrix}$

$\mathrm{Null}(A) = \mathrm{span}(\underline{v}_1, \underline{v}_2, \underline{v}_3)$
$= \mathrm{span}(\underline{v}_1, \underline{v}_2)$

$\dim \mathrm{Null}(A) = \underline{\underline{2}}$

$\mathrm{rk}\, A = \underline{\underline{2}}$

Correct Answer: C

$\perp$

$$\begin{pmatrix} 1 & 7 & 4 \\ 1 & 0 & -3 \\ 0 & 1 & 1 \\ 1 & 4 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 7 & 4 \\ 0 & -7 & -7 \\ 0 & 1 & 1 \\ 0 & -3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 7 & 4 \\ 0 & 1 & 1 \\ 0 & -7 & -7 \\ 0 & -3 & -3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 7 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

[E] 4.6:

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow$

Char. eqn:

$$\boxed{-\lambda^3 + c_1 \cdot \lambda^2 - c_2 \lambda + c_3 = 0}$$

$c_1 = \mathrm{tr}(A) = a_{11} + a_{22} + a_{33}$

$c_2 = M_{12,12} + M_{23,23} + M_{13,13}$

$c_3 = \det(A)$

a) $A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & -1 & 1 \\ 3 & 3 & 3 \end{pmatrix}$:

$-\lambda^3 + 3\lambda^2 - (-9 - 6 - 3)\lambda + 0 = 0$

$-\lambda^3 + 3\lambda^2 + 18\lambda = 0$

$|A| = 1 \cdot (-6) - 4 \cdot 3 + 2 \cdot 9 = 0$

$-\lambda(\lambda^2 - 3\lambda - 18) = 0$

$\underline{\lambda = 0}$ or $\lambda = \dfrac{3 \pm \sqrt{9 + 72}}{2}$

$= \dfrac{3 \pm 9}{2}$

$\lambda = \underline{6} \quad \lambda = \underline{-3}$