Key Problems

Problem 1.
Find the equilibrium states and determine their stability. Sketch the solution curve $y = y(t)$.

    a) $y' = 6 - 2y$ b) $y' = y^2 - 4$ c) $y' = 5y(1 - y/10)$

Problem 2.
Solve the differential equations:

    a) $y'' + 6y' - 16y = 16t - 22$ b) $y'' + 6y' + 9y = 4e^{-t}$

    c) $y'' - 3y' + 2y = 3e^{2t}$ d) $y'' - y = t^2$

Problem 3.
Show that there are separable, non-linear differential equations with the following functions as particular solutions:

    a) $y = \sqrt{t^2 - 3}$ b) $y = \dfrac{2}{1 - t^2}$

Exercise Problems

Textbook problems:	[E] 7.24 - 7.34
Exam problems	[Final 12/2015] Q2, [Final 11/2017] Q2, [Final 11/2018] Q2ac, Q5,
	[Final 01/2021] Q3bc

Answers to Key Problems

Problem 1.
    a) $y_e = 3$ is globally asymptotically stable

    b) $y_e = -2$ is stable (but not globally asymptotically stable), $y_e = 2$ is unstable

    c) $y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)

Problem 2.
    a) $y = C_1 e^{-8t} + C_2 e^{2t} + 1 - t$ b) $y = C_1 e^{-3t} + C_2 t e^{-3t} + e^{-t}$

    c) $y = C_1 e^{2t} + C_2 e^t + 3t e^{2t}$ d) $y = C_1 e^t + C_2 e^{-t} - t^2 - 2$