Key Problems

Problem 1.
Determine the definiteness of the symmetric matrix:

a) $A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$ c) $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$

d) $A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$ e) $A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$

Problem 2.
Find the symmetric matrix of the quadratic form, and determine its definiteness:

a) $f(x,y) = x^2 - 8xy + 3y^2$ b) $f(x,y,z) = 2x^2 - 2xz + 3y^2 + z^2$

c) $f(x,y,z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$ d) $f(x,y,z,w) = xw - yz$

Problem 3.
Determine the definiteness of the symmetric matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

Problem 4.
Find an orthogonal matrix P such that $P^TAP = D$ is a diagonal matrix, and write down the quadratic form $\mathbf{x}^TA\mathbf{x}$ in the the coordinates $\mathbf{u} = P^T\mathbf{x}$:

a) $A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$ b) $A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$ c) $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$

Problem 5.
Check that the Markov chain with transition matrix A is regular, and find its equilibrium state $\mathbf{v}$:

a) $A = \begin{pmatrix} 0.40 & 0.15 \\ 0.60 & 0.85 \end{pmatrix}$ b) $A = \begin{pmatrix} 0.77 & 0.46 \\ 0.23 & 0.54 \end{pmatrix}$ c) $A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$

d) $A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$ e) $A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$ f) $A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$

Exercise Problems

Problems from the textbook:	[E] 4.8 - 4.17
Exam problems:	Midterm exam 10/2018 Question 1-7
	Final exam 01/2023 Question 1, 3a

Answers to Key Problems

Problem 1.

a) Positive definite

b) Negative definite

c) Positive definite

d) Positive semi-definite

e) Indefinite

Problem 2.

a) Indefinite

b) Positive definite

c) Positive semi-definite

d) Indefinite

Problem 3.

Positive semi-definite

Problem 4.

a) $P = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

$2u^2 + 4v^2$

b) $P = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$

$3u^2 + 4v^2 + 5w^2$

c) $P = \frac{1}{\sqrt{6}}\begin{pmatrix} -\sqrt{3} & -1 & \sqrt{2} \\ 0 & 2 & \sqrt{2} \\ \sqrt{3} & -1 & \sqrt{2} \end{pmatrix}$

$-u^2 - v^2 + 2w^2$

Problem 5.

a) $\mathbf{v} = \begin{pmatrix} 1/5 \\ 4/5 \end{pmatrix}$

b) $\mathbf{v} = \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix}$

c) $\mathbf{v} = \begin{pmatrix} 3/17 \\ 14/17 \end{pmatrix}$

d) $\mathbf{v} = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$

e) $\mathbf{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$

f) $\mathbf{v} = \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix}$