Key Problems

Problem 1.
Check if the given sets are compact (closed and bounded). It is useful to sketch the sets:

a) $D = \{(x,y) : x,y \geq 0 \text{ and } 2x + 3y \leq 6\}$
b) $D = \{(x,y) : 4x^2 + 9y^2 \leq 36\}$
c) $D = \{(x,y) : x,y \geq 1 \text{ and } 2x + 3y \geq 12\}$
d) $D = \{(x,y) : 4xy \leq 1 \text{ and } x,y > 0\}$

Problem 2.
Solve the Lagrange problems:

a) $\max f(x,y,z) = x + 2y + 3z$ when $2x^2 + y^2 + 2z^2 = 9$
b) $\max / \min f(x,y,z) = x^2 + y^2 + z^2$ when $3x^2 + 2y^2 + 2z^2 = 12$

Problem 3.
Use the second order condition to solve the Lagrange problem:

a) $\max / \min f(x,y,z) = 4x^2 + 9y^2 + z^2$ when $x + y + z = 1$
b) $\max / \min f(x,y,z,w) = xw - yz$ when $x^2 + 4y^2 = 4$ and $4z^2 + 9w^2 = 36$

Problem 4.
Determine if there are any admissible points such that the NDCQ fails when the constraints are given by:

a) $xyz = 1$
b) $3x^2 + 3y^2 + 8z^2 = 1$
c) $x^3 + y^3 + z^3 = 0$
d) $xy - zw = 1$ and $x + y + z + w = 4$

Exercise Problems

Problems from the textbook:	[E] 6.1, 6.2, 6.3ab, 6.4, 6.11
Exam problems	Final exam 11/2019 Question 4ab
	Final exam 11/2022 Question 2
	Final exam 11/2023 Question 1d, Question 3

Answers to Key Problems

Problem 1.
a) Compact
b) Compact
c) Not compact (not bounded)
d) Not compact (not bounded)

Problem 2.
a) $f_{\max} = 9$
b) $f_{\max} = 6,\ f_{\min} = 4$

Problem 3.
a) $f_{\min} = 36/49$
b) $f_{\max} = 4,\ f_{\min} = -4$

Problem 4.
a) None
b) None
c) $(x,y,z) = (0,0,0)$
d) None