Key Problems

Problem 1.
Solve the Kuhn-Tucker problems:

a) $\max f(x,y) = 3x - 4y$ when $x^2 + y^2 \leq 1$
b) $\max f(x,y,z) = 3x^2 - y^2 - 2z^2$ when $2x^4 + 2y^4 + z^4 \leq 18$
c) $\max f(x,y,z,w) = x + 4y + 2z + 5w$ when $2x^2 + 2y^2 + 2yz + 2z^2 + 2w^2 \leq 21$

Problem 2.
We consider the Kuhn-Tucker problem $\min f(x,y) = x^2 + y^2$ when $xy \geq 4$.

a) Write the Kuhn-Tucker in standard form, and find candidates for minimum.
b) Sketch the set of admissible points. Is it compact?
c) Find the minimum value in the Kuhn-Tucker problem, if it exists.

Problem 3.
Solve the Kuhn-Tucker problems. Difficult problems!

a) $\max f(x,y,z,w) = xz + yw$ when $x^2 + y^2 \leq 1$ and $4z^2 + 9w^2 \leq 36$
b) $\max f(x,y) = \ln(x^2 y) - x - y$ when $x \geq 1$, $y \geq 1$ and $x + y \geq 4$

Problem 4.
Determine the range of f. Review from Lecture 6 / unconstrained optimization!

a) $f(x,y,z) = 2\ln(u)/u^2$ where $u = 1 + x^2 + 4xy + 5y^2 - 2yz + 8z^2$
b) $f(x,y,z) = 1/u$ where $u = x^2 + 2xy + 4xz + 3y^2 + 8yz + 7z^2 - 2x - 2y - 2z + 4$

Exercise Problems

Problems from the textbook: [E] 6.3cd, 6.5 - 6.7, 6.10

Answers to Key Problems

Problem 1.
a) $f_{\max} = 5$ b) $f_{\max} = 9$ c) $f_{\max} = 21$

Problem 2.
a) $\max -f(x,y) = -x^2 - y^2$ when $-xy \leq -4$, $(2,2;2), (-2,-2;2)$
b) No c) $f_{\min} = 8$

Problem 3.
a) $f_{\max} = 3$ b) $f_{\max} = 8\ln 2 - 3\ln 3 - 4$

Problem 4.
a) $[0, 1/e]$ b) $(0, 1/2]$