QUESTION 1.

We consider the function $f(x, y, z, w) = x^5 + xy^2 - zw$.

(a) Find all stationary points of f.
(b) Compute the Hessian matrix of f. Classify the stationary points of f as local maxima, local minima or saddle points.

QUESTION 2.

We consider the matrix A and the vector $\mathbf{v}$ given by

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & s & s^2 \\ 1 & -1 & 1 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$$

(a) Compute the determinant and the rank of A.
(b) Find all values of s such that $\mathbf{v}$ is an eigenvector for A.
(c) Compute all eigenvalues of A when $s = -1$. Is A diagonalizable when $s = -1$?

QUESTION 3.

(a) Solve the difference equation $x_{t+1} = 3x_t + 4$, $x_0 = 2$ and compute x_5.
(b) Find the general solution of the differential equation $y'' + 2y' - 35y = 11e^t - 5$.
(c) Solve the initial value problem $(2t + y) - (4y - t)y' = 0$, $y(0) = 0$.

QUESTION 4.

We consider the function $f(x, y) = xye^{x+y}$ defined on $D_f = \{(x, y) : (x + 1)^2 + (y + 1)^2 \leq 1\}$.

(a) Compute the Hessian of f. Is f a convex function? Is f a concave function?
(b) Find the maximum and minimum values of f.