BI NORWEGIAN BUSINESS SCHOOL

Written examination:	**GRA 60353 Mathematics**	
Examination date:	13.12.2013 09:00 – 12:00	Total no. of pages: 2
Permitted examination	A bilingual dictionary and BI-approved calculator TEXAS	
support material:	INSTRUMENTS BA II Plus	
Answer sheets:	Squares	
	Counts 80% of GRA 6035	The subquestions have equal weight
Ordinary exam		Responsible department: Economics

All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 4(d) can be skipped, but gives 6p extra credit if answered correctly.

Question 1.

Let f be the function given by $f(x, y, z, w) = xw - yz$.

(a) **(6p)** Compute the partial derivatives and the Hessian matrix of f.
(b) **(6p)** Find all stationary points of f, and classify them as local max, local min or saddle points.
(c) **(6p)** Does f have a global maximum? Justify your answer.

Question 2.

We consider the matrix A given by

$$A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

(a) **(6p)** Compute the rank of $A + I$ and $A - I$, where I is the identity matrix.
(b) **(6p)** Is A diagonalizable? Justify your answer.
(c) **(6p)** Find all eigenvectors of A with eigenvalue $\lambda = -1$.

Question 3.

Solve the difference equation:

(a) **(6p)** $y_{t+2} - 11y_{t+1} + 28y_t = 36t + 18$

Solve the differential equations:

(b) **(6p)** $y' = 4y + te^t$
(c) **(6p)** $\dfrac{y}{y^2 + t^2 + 3}\, y' + \dfrac{t}{y^2 + t^2 + 3} = 0,$ with initial condition $y(1) = 2$

Question 4.

We consider the following Kuhn-Tucker problem:

$$\max\ f(x,y,z,w) = xw - yz \text{ subject to } \begin{cases} x^2 + y^2 \leq 1 \\ 4z^2 + 9w^2 \leq 36 \end{cases}$$

(a) **(6p)** Write down the Kuhn-Tucker conditions. Show that there is a solution of these conditions with $(x, y, z, w) = (0, 1, -3, 0)$, and find the corresponding multipliers.

(b) **(6p)** Show that $(x, y, z, w) = (0, 1, -3, 0)$ solves the Kuhn-Tucker problem.

(c) **(6p)** Estimate the maximum value in the Kuhn-Tucker problem we obtain when we replace the second constraint with $4.2z^2 + 9w^2 \leq 36$.

(d) **Extra credits (6p)** Find all five solutions of the Kuhn-Tucker conditions in a).