| **Written examination:** | **GRA 60353** **Mathematics** | |
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| Examination date: | 12.05.2014    09:00 – 12:00   Total no. of pages:   2 | |
| Permitted examination | A bilingual dictionary and BI-approved calculator TEXAS | |
| support material: | INSTRUMENTS BA II Plus | |
| Answer sheets: | Squares | |
| | Counts 80% of GRA 6035 | The subquestions have equal weight |
| Re-take exam | | Responsible department: Economics |

**All subquestions have the same weight and give maximal score 6p each. Answers to the first 12 subquestions give a maximal score of 72p (100%). Question 4(d) can be skipped, but gives 6p extra credit if answered correctly.**

QUESTION 1.

We consider the matrix $A$ given by

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

(a) **(6p)** Compute the ranks of $A$ and of $A + I$, where $I$ is the identity matrix.
(b) **(6p)** Find the eigenvalues of $A$.
(c) **(6p)** Is $A$ diagonalizable? Justify your answer.

QUESTION 2.

Let $f$ be the function given by $f(x, y, z, w) = x^3 + 3xy^2 - 3x - 2z^3 + 6zw^2 - 3w$.

(a) **(6p)** Compute the partial derivatives and the Hessian matrix of $f$.
(b) **(6p)** Find all stationary points of $f$, and classify them as local max, local min or saddle points.
(c) **(6p)** Is $f$ concave?

QUESTION 3.

Solve the difference equation:

(a) **(6p)** $y_{t+1} - 2y_t = 3t$

Solve the differential equations:

(b) **(6p)** $y'' - 12y' + 20y = 2te^t$
(c) **(6p)** $y' + \ln(t)\, y = \ln(t)$ when $t > 0$

QUESTION 4.

We consider the following Kuhn-Tucker problem:
$$\max \ f(x,y,z) = x^3 + y^3 + z^3 - 3xyz \quad \text{subject to} \quad x^3 + y^3 + z^3 \leq 8$$

(a) **(6p)** Write down the Kuhn-Tucker conditions, and find all solutions of the Kuhn-Tucker conditions with $\lambda = 1$.

(b) **(6p)** Show that the NDCQ is satisfied at all admissible points.

(c) **(6p)** Is the set of admissible points bounded? Justify your answer.

(d) **Extra credits (6p)** Determine if the Kuhn-Tucker problem has a solution or not, and find the solution if it exists.