EXAMINATION QUESTION PAPER - Written examination

GRA 60353
Mathematics

Department of Economics

Start date:	27.11.2019	Time 09.00
Finish date:	27.11.2019	Time 12.00

Weight:	80% of GRA 6035
Total no. of pages:	2 incl. front page
Answer sheets:	Squares
Examination support materials permitted:	BI-approved exam calculator. Simple calculator. Bilingual dictionary.

Exam	Final exam in GRA 6035 Mathematics
Date	November 27th, 2019 at 0900 - 1200

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the matrix A, and the column vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ of A, given by

$$A = \begin{pmatrix} 2 & 1 & 5 & 9 \\ -1 & 1 & 2 & -3 \\ 3 & 0 & 1 & 10 \\ 0 & 3 & 0 & -6 \end{pmatrix}, \quad \mathbf{v}_1 = \begin{pmatrix} 2 \\ -1 \\ 3 \\ 0 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 5 \\ 2 \\ 1 \\ 0 \end{pmatrix}, \quad \mathbf{v}_4 = \begin{pmatrix} 9 \\ -3 \\ 10 \\ -6 \end{pmatrix}$$

(a) **(6p)** Compute the rank of A.
(b) **(6p)** Find $\dim \text{Null}(A)$ and a base for $\text{Null}(A)$.
(c) **(6p)** If possible, express $\mathbf{v}_4$ as a linear expression of the vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

Question 2.
We consider the matrix A and the vector $\mathbf{v}$ given by

$$A = \begin{pmatrix} 4 & 0 & 6 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} -3 \\ -1 \\ 2 \end{pmatrix}$$

(a) **(6p)** Show that $\mathbf{v}$ is an eigenvector of A.
(b) **(6p)** Find the eigenvalues of A.
(c) **(6p)** Determine whether A is diagonalizable.

Question 3.

(a) **(6p)** Solve the differential equation $y' - 4y = 10e^{-t}$.
(b) **(6p)** Solve differential equation $2t + 2ty^2 + (2y + 2yt^2)y' = 0$.
(c) **(6p)** Solve the linear system of differential equations:

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = \begin{pmatrix} 4 & 0 & 6 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

Question 4.
We consider the Lagrange problem given by

$$\min \ f(x, y, z) = x^2 + y^2 + z^2 - xy + xz - yz \text{ subject to } x + y + z = 11$$

(a) **(6p)** Determine whether f is convex or concave.
(b) **(6p)** Solve the Lagrange problem, and find the minimum value.
(c) **(6p)** Use the envelope theorem to estimate the minimum value of the Lagrange problem

$$\min \ f(x, y, z) = x^2 + y^2 + z^2 - xy + xz - yz \text{ subject to } x + y + z = 10$$

Question 5.
Extra credit (6p) Find the particular solution of the system of difference equations that satisfies the given initial condition:

$$\mathbf{y}_{t+1} = \begin{pmatrix} 4 & 0 & 6 \\ -1 & 3 & 0 \\ 1 & 1 & 2 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$