EXAMINATION QUESTION PAPER - Take-home examination

GRA 00351
Mathematics

Department of Economics

Start date:	18.01.2021	Time 09.00
Finish date:	18.01.2021	Time 12.00

Weight:	100% of GRA 0035
Total no. of pages:	2 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf

Exam	Final exam in GRA 6035 Mathematics
Date	January 18th, 2021 at 0900 - 1200

This exam consists of 11 problems. **You must give reasons for all your answers**. To get full score, your answers should be **short, clear, and precise**.

- **You must hand in your exam papers as a single PDF file. It must be handwritten.**
- The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.
- All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

Question 1.
We consider the matrix given by

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 \\ 2 & 4 & 2 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 6 & 10 \end{pmatrix}$$

(a) **(6p)** Find a base of $\mathrm{Null}(A)$, and determine the eigenvalues of A.
(b) **(6p)** Find a base of $\mathrm{Col}(A)$. If possible, find a vector in $\mathbb{R}^4$ not in $\mathrm{Col}(A)$.
(c) **(6p)** Determine the definiteness of the quadratic form $f(\mathbf{x}) = \mathbf{x}^T A\,\mathbf{x}$.
(d) **(6p)** Explain how to use eigenvalues and eigenvectors to find the maximal and minimal value of $f(\mathbf{x}) = \mathbf{x}^T A\,\mathbf{x}$ on $D = \{\mathbf{x} : \mathbf{x}^T\mathbf{x} = 1\}$. You should lay out a procedure for how to do this, and express the answers in terms of eigenvalues and eigenvectors (numerical values are not necessary).

Question 2.

(a) **(6p)** Solve $\max f(x,y,z) = \ln(5 - x^2 + xy - y^2 + yz - z^2 - xz)$.
(b) **(6p)** Determine whether $D = \{(x,y,z,w) : xw + yz \leq -2\} \subseteq \mathbb{R}^4$ is a compact set.
(c) **(6p)** Find all points satisfying the Kuhn-Tucker conditions, and the value of the objective function at each of these points:

$$\min f(x,y,z,w) = x^2 + 4y^2 + 9z^2 + w^2 \text{ subject to } xw + yz \leq -2$$

Question 3.

(a) **(6p)** Solve the difference equation $y_{t+2} - y_{t+1} - 2y_t = 4t$, and find y_{17} when $y_0 = y_1 = 1$.
(b) **(6p)** Determine whether $t^2y' + 2ty = 1$ is (i) separable, (ii) linear, (iii) exact. Use this to solve the differential equation in at least two different ways.
(c) **(6p)** Find a linear second order differential equation with $y = 3e^{-2t} - 5e^{t} + 12e^{-3t}$ as solution.
(d) **(6p)** Find a 3×3 matrix A such that $\mathbf{y}' = A\mathbf{y}$ has a solution $\mathbf{y} = (y, y', y'')$ with y as in (c).