EXAMINATION QUESTION PAPER - Take-home examination

GRA 00351
Mathematics

Department of Economics

Start date:	19.03.2021	Time 09.00
Finish date:	19.03.2021	Time 12.15

Weight:	100% of GRA 0035
Total no. of pages:	2 incl. front page
No. of attachments files to question paper:	0
To be answered:	Individually

Answer paper size:	No limit. excl. attachments
Max no. of answer paper attachment files:	0
Allowed answer paper file types:	pdf

Exam	Final exam in GRA 6035 Mathematics
Date	March 19th, 2021 at 0900 - 1215

This exam consists of 11 problems. **You must give reasons for all your answers**. To get full score, your answers should be **short, clear, and precise**.

- **You must hand in your exam papers as a single PDF file. It must be handwritten.**
- The answer paper must be written and prepared individually. Collaboration with others is not permitted and is considered cheating.
- All answer papers are automatically subjected to plagiarism control. Students may also be called in for an oral consultation as additional verification of an answer paper.

Question 1.
We consider the matrix given by

$$A = \begin{pmatrix} 1 & -1 & 0 & 4 \\ 3 & 2 & 1 & 0 \\ 2 & 1 & 1 & 0 \\ 0 & -2 & 0 & 4 \end{pmatrix}$$

(a) **(6p)** Determine whether the column vectors of A are linearly independent, and express the third column vector as a linear combinations of the others if it is possible.
(b) **(6p)** Find $\dim \text{Null}(A)$, and determine whether the vector $\mathbf{w} = (1, 1, -5, 0)$ is in $\text{Null}(A)$.
(c) **(6p)** Find the stationary points of the function $f(\mathbf{x}) = \mathbf{x}^T A\,\mathbf{x}$ and determine their type.
(d) **(6p)** Is it true that $f(\mathbf{x}) = \mathbf{x}^T M\mathbf{x}$ is positive definite for any $n \times n$ matrix M (not necessarily symmetric) with n positive eigenvalues? Explain why, or give a counterexample.

Question 2.
We consider the quadratic form $Q(x, y, z) = 7x^2 + 8xy + 4xz + 13y^2 - 2yz + z^2$.

(a) **(6p)** Find the maximum value of $f(x, y, z) = \ln(u)/u^3$ where $u = Q(x, y, z) + 2$.
(b) **(6p)** We consider a Kuhn-Tucker problem with constraint $x^2+y^2+z^2 \leq 5$. Determine whether the set of admissible points is compact, and whether all admissible points satisfy the NDCQ.
(c) **(6p)** Solve the Kuhn-Tucker problem, and find the maximum value if it exists:

$$\max Q(x, y, z) \text{ when } x^2 + y^2 + z^2 \leq 5$$

(d) **(6p)** Estimate the maximum value of the Kuhn-Tucker problem

$$\max 8x^2 + 8xy + 4xz + 13y^2 - 2yz + z^2 \text{ when } x^2 + y^2 + z^2 \leq 5$$

Question 3.

(a) **(6p)** Solve the difference equation $y_{t+2} - 7y_{+1} + 6y_t = -4 \cdot 2^t,\ y_1 = 9,\ y_3 = 225$.
(b) **(6p)** Solve the differential equation $y' + y - 1 = t(y - 1)$ as a linear and as a separable differential equation, and find $y(2)$ when $y(0) = 4$.
(c) **(6p)** Solve the system of linear differential equations $\mathbf{y}' = A \cdot \mathbf{y}$ when A is the matrix

$$A = \begin{pmatrix} 4 & -1 & 2 \\ 1 & 1 & -1 \\ 2 & -1 & 4 \end{pmatrix}$$