Exam	Final exam in GRA 6035 Mathematics
Date	January 28th, 2022 at 1300 - 1600

This exam consists of 12+1 problems (one additional problem is for extra credits, and can be skipped). Each problem has a maximal score of 6p, and 72p (12 solved problems) is marked as 100% score.

You must give reasons for your answers. Precision and clarity will be emphasized when evaluating your answers.

Question 1.
We consider the matrix A given by

$$A = \begin{pmatrix} 3 & 0 & 0 & 1 \\ 0 & 2 & 4 & 4 \\ -1 & 0 & -2 & -5 \\ 1 & 0 & 0 & 3 \end{pmatrix}$$

(a) **(6p)** Compute the determinant of A.
(b) **(6p)** Determine the dimension of the column space and the null space of A.
(c) **(6p)** Show that $\lambda = 2$ is an eigenvalue of A, and find a base of the eigenspace E_2.
(d) **(6p)** Find all eigenvalues of A.

Question 2.
(a) **(6p)** Solve the difference equation $6y_{t+2} + y_{t+1} - y_t = 6t + 1$.
(b) **(6p)** Solve the differential equation $ty' - 2y = t^2$.
(c) **(6p)** Solve the differential equation $y^2 - 3t^2y + (2ty - t^3)y' = 0$.
(d) **(6p)** Solve the system of difference equations:

$$\mathbf{y}_{t+1} = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}_t$$

Question 3.
We consider the function f given by $f(x, y, z) = 6x + 6y + 6z - x^2 - 4z^2 - 2xy - 4yz$ and the Kuhn-Tucker problem given by

$$\max\ f(x, y, z) \text{ subject to } g(x, y, z) = 2x^2 + y^2 + 3z^2 + 8xz \leq 9$$

(a) **(6p)** Find all stationary points of f and classify them.
(b) **(6p)** Write down the Kuhn-Tucker conditions of the Kuhn-Tucker problem.
(c) **(6p)** Find all points $(x, y, z; \lambda)$ with $\lambda = 1$ that satisfy the Kuhn-Tucker conditions.
(d) **(6p)** Solve the Kuhn-Tucker problem.

Let $D = \{(x, y, z) : g(x, y, z) \leq 9\}$ be the set of admissible points in the Kuhn-Tucker problem.

(e) **Extra credit (6p)** Determine whether D is a compact set.