Exam	Final exam in GRA 6035 Mathematics
Date	January 9th 2023 at 0900 - 1200

You must give reasons for your answers.

Question 1.
We consider the matrix A and the vector $\mathbf{v}$ given by

$$A = \begin{pmatrix} 12 & 6 & -3 \\ 6 & 3 & 6 \\ -3 & 6 & -4 \end{pmatrix}, \quad \mathbf{v} = \begin{pmatrix} 1 \\ -2 \\ a \end{pmatrix}$$

(a) **(6p)** Compute the determinant of A.
(b) **(6p)** Determine the dimension of the column space of A.
(c) **(6p)** Determine the values of a and λ such that $\mathbf{v}$ is in the eigenspace E_λ of A.
(d) **(6p)** Find all eigenvalues of A.

Question 2.

(a) **(6p)** Solve the difference equation: $y_{t+2} - 2y_{t+1} - 8y_t = -9t$
(b) **(6p)** Solve the differential equation: $y' = 2t - 4ty$
(c) **(6p)** Solve the system of differential equations: $u' = v,\ v' = 8u + 2v + 24$
(d) **(6p)** Find the particular solution of $ty' = e^{-y}$ for $t > 0$ satisfying the initial condition $y(1) = 0$.

Question 3.
Let f be the quadratic form given by $f(x, y, z, w) = 2x^2 - 2xy + 6xz + y^2 + 10z^2 + 2zw + w^2$, and consider the Lagrange problem

$$\min f(x, y, z, w) = 2x^2 - 2xy + 6xz + y^2 + 10z^2 + 2zw + w^2 \text{ when } 3x + y + 8z - 4w = 33$$

(a) **(6p)** Determine the definiteness of the quadratic form f.
(b) **(6p)** Find all points $(x, y, z, w; \lambda)$ with $w = -4$ that satisfy the Lagrange conditions.
(c) **(6p)** Find the minimum value in the Lagrange problem, if it exists.
(d) **(6p)** Find the range of the function $p(x, y, z, w) = u^2 - 4u + 7$, where $u = f(x, y, z, w)$.