Exam	Final exam in GRA 6035 Mathematics
Date	November 24th 2025 at 0900 - 1400

You must give reasons for your answers.

Question 1.

(a) **(3p)** Find the general solution of the differential equation $y'' + 4y' - 21y = 0$.

(b) **(3p)** Find the equilibrium state of the Markov chain with transition matrix

$$A = \begin{pmatrix} 0.76 & 0.24 \\ 0.24 & 0.76 \end{pmatrix}$$

(c) **(3p)** Determine the definiteness of $f(x, y, z) = x^2 + 4xy + 6xz + 4y^2 + 12yz + 10z^2$.

(d) **(3p)** Define an orthogonal matrix. Show that if A is orthogonal, then $|A| = 1$ or $|A| = -1$.

Question 2.

We consider the matrix A given by

$$A = \begin{pmatrix} -3 & 2 & 0 \\ -2 & 0 & 2 \\ 0 & -2 & 3 \end{pmatrix}$$

(a) **(6p)** Compute the determinant and trace of A.

(b) **(6p)** Find a base of the nullspace of A.

(c) **(6p)** Find the eigenvalues of A, and determine whether A is diagonalizable.

Question 3.

We consider the function u given by

$$u(x, y, z) = 3x^2 - 2xy + 4xz + 2y^2 + 2yz + 4z^2 - 6x - 8y - 16z + 21$$

(a) **(6p)** Find the stationary points of u.

(b) **(6p)** Show that u has a minimum, and find the minimum value of u.

(c) **(6p)** Determine the range of the function $f(x, y, z) = u/\ln u$, where $u = u(x, y, z)$.

Question 4.

(a) **(6p)** Find the general solution $\mathbf{y} = (y_1, y_2)$ of the linear system of differential equations:

$$\mathbf{y}' = \begin{pmatrix} 0 & 1 \\ 21 & -4 \end{pmatrix} \cdot \mathbf{y} + \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

(b) **(6p)** Solve the difference equation $y_{t+1} - y_t = 1 + 4t$.

(c) **(6p)** Solve the initial value problem $2t - 2ty + (2y - t^2)y' = 0,\ y(1) = 0$.

Question 5.

We consider the Kuhn-Tucker problem

$$\max f(x, y, z, w) = x + 2y + 4z + w \text{ when } 3x^2 + y^2 + 4z^2 + w^2 - 2xw \leq 11$$

(a) **(6p)** Find all candidate points $(x, y, z, w; \lambda)$ that satisfies the Kuhn-Tucker conditions.

(b) **(6p)** Show that the problem has a maximum, and find the maximum value.

(c) **(6p)** Estimate the maximum value of the Kuhn-Tucker problem

$$\max f(x, y, z, w) = x + 2y + 4z + w \text{ when } 3x^2 + y^2 + 4z^2 + w^2 - xw \leq 11$$