NORWEGIAN BUSINESS SCHOOL   Department of Economics

Solutions:	**GRA 60352 Mathematics**
Examination date:	24.09.2010, 14:00 − 15:00
Permitted examination aids:	Bilingual dictionary
	BI-approved exam calculator: Texas Instruments BA II Plus$^{\text{TM}}$
Answer sheets:	Answer sheet for multiple choice examinations
Total number of pages:	4
Number of attachments:	1 (example of how to use the answer sheet)

PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.

- Students must provide the following information on the answer sheet:
  - Examination code
  - Personal initials
  - ID number

  The student registration number must be recorded with both the appropriate numbers and by putting an "X" by the corresponding number in the columns below.

- Pens with green ink and pencils cannot be used in filling in answer sheets. Answer sheets must not be used for writing rough drafts.

- **All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.**

- **Cancel an "X" by filling in the box completely (boxes that are completely filled in will not be registered). "X" in two boxes for one question will be registered as a wrong answer.**

- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An "X" under E indicates that you choose not to answer question 5.

- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**

- There is only <u>one</u> right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the simplest questions first.

- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.

- You can keep the examination papers.

Good luck!

Correct answers: D-D-C-D-C-A-A-B

<div align="center">QUESTION 1.</div>

Since the augmented matrix of the system is in reduced echelon form, we see that the system is consistent and has two free variables, x_3 and x_5. Hence the correct answer is alternative $\boxed{D}$. This question can also be answered using minors.

<div align="center">QUESTION 2.</div>

We compute the determinant
$$\begin{vmatrix} 2 & 1 & 0 \\ 3 & 2 & 1 \\ -1 & 1 & h \end{vmatrix} = h - 3$$
Hence the vectors are linearly independent exactly when $h \neq 3$, and the correct answer is alternative $\boxed{D}$. This question can also be answered using Gauss elimination.

<div align="center">QUESTION 3.</div>

We compute an echelon form of A using elementary row operations, and get
$$A = \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & -2 & 4 \end{pmatrix} \dashrightarrow \begin{pmatrix} 2 & 5 & -3 & -4 & 8 \\ 0 & -3 & 2 & 5 & -7 \\ 0 & 0 & 0 & 0 & -6 \end{pmatrix}$$
Hence A has rank 3, and the correct answer is alternative $\boxed{C}$. This question can also be answered using minors. For instance, the minor of order 3 obtained by deleting column 3 and 4 is non-zero.

<div align="center">QUESTION 4.</div>

The characteristic equation of A is $\lambda^2 - 9 = 0$. Hence the eigenvalues of A is $\lambda = \pm 3$, and the correct answer is alternative $\boxed{D}$.

<div align="center">QUESTION 5.</div>

We solve the linear system $(A - 7I)\mathbf{v} = \mathbf{0}$, and see that the system has one degree of freedom. Hence $\lambda = 7$ is an eigenvalue. This is the only eigenvalue of A, since the characteristic equation of A is $\lambda^2 - 14\lambda + 49 = 0$ and has 7 as a double root. Hence the correct answer is alternative $\boxed{C}$.

<div align="center">QUESTION 6.</div>

The symmetric matrix associated with Q is $A = \left(\begin{smallmatrix} 1 & -2 \\ -2 & 4 \end{smallmatrix} \right)$, and we compute its eigenvalues to be 0 and 5. Hence the correct answer is alternative $\boxed{A}$. This question can also be answered using the fact that $Q(x_1, x_2) = (x_1 - 2x_2)^2$.

<div align="center">QUESTION 7.</div>

The function f is a sum of a linear function and a quadratic form with symmetric matrix
$$A = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
Since A has eigenvalues $\lambda = 1, 2, 4$, the quadratic form is positive definite and therefore convex (but not concave). Hence the correct answer is alternative $\boxed{A}$.

We solve the equation $A\mathbf{v} = \mathbf{v}$, which can be written as a linear system

$$\begin{pmatrix} 0.97 - 1 & 0.02 \\ 0.03 & 0.98 - 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.03 & 0.02 \\ 0.03 & -0.02 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

This system has one free variable, and the solutions can be written as $x = 2t$, $y = 3t$, with $x + y = 5t$. Hence there is a steady state with

$$\frac{x}{x+y} = \frac{2t}{5t} = 0.4 = 40\%$$

of the cars at the downtown location. The correct answer is therefore alternative $\boxed{B}$.