BI NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352** Mathematics		
Examination date:	19.04.2012	09:00 – 10:00	Total no. of pages: 2
			No. of attachments: 0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus		
Answer sheets:	Answer sheet for multiple-choice examinations		
	Counts 20% of GRA 6035		The questions are weighted equally
Re-sit exam			Responsible department: Economics

Correct answers: B-A-D-C-A-D-C-B

Question 1.

We reduce the augmented matrix to echelon form:

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 4 & -4 \\ 0 & 1 & 3 & 9 & 2 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 3 & -7 \\ 0 & 0 & 2 & 8 & -1 \end{array}\right) \dashrightarrow \left(\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 0 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 1 & 3 & -7 \\ 0 & 0 & 0 & 2 & 13 \end{array}\right)$$

From the pivot positions, we see that the system has a unique solution. The correct answer is alternative **B**.

Question 2.

We form the matrix A with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and reduce A to an echelon form:

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 2 & 8 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 2 \end{pmatrix}$$

From the pivot positions, we see that $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent. Hence the correct answer is alternative **A**.

Question 3.

We reduce the matrix A to an echelon form:

$$A = \begin{pmatrix} 1 & 2 & -2 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & 6 & 1 & h-1 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & -6 & 13 & h-7 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 2 & -2 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 7 & h-7 \end{pmatrix}$$

We see that the rank of A is three for all values of h, and the correct answer is alternative **D**.

Question 4.

The characteristic equation of A is $\lambda^2 - 10\lambda + 25 = 0$, and therefore there is only one (double) eigenvalue $\lambda = 5$. The correct answer is alternative **C**.

Question 5.

The eigenvalues of A are $\lambda = 1$ (with multiplicity two) and $\lambda = -1$, since we have

$$\det(A - \lambda I) = \begin{pmatrix} 1-\lambda & h & -2h \\ 0 & -1-\lambda & 4 \\ 0 & 0 & 1-\lambda \end{pmatrix} = (1-\lambda)^2(-1-\lambda) = 0$$

We compute the eigenvectors of $\lambda = 1$, the eigenvalue of multiplicity 2, by reducing the matrix $A - I$ to an echelon form:

$$\begin{pmatrix} 0 & h & -2h \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & -2 & 4 \\ 0 & h & -2h \\ 0 & 0 & 0 \end{pmatrix} \dashrightarrow \begin{pmatrix} 0 & -2 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

We see that there are two degrees of freedom for all values of h. Therefore, A is diagonalizable for all values of h and the correct answer is alternative **A**.

Question 6.

The symmetric matrix of the quadratic form $Q(x_1, x_2) = 4x_1^2 - 15x_1x_2 + 36x_2^2$ is

$$A = \begin{pmatrix} 4 & -15/2 \\ -15/2 & 36 \end{pmatrix}$$

The leading principal minors are $D_1 = 4$ and $D_2 = 4 \cdot 36 - (-15/2)^2 > 0$. Therefore A is positive definite, and the correct answer is alternative **D**.

Question 7.

We compute the Hessian matrix of $f(x, y, z) = \ln(x + y + z)$: First, we compute the first order partial derivatives

$$f'_x = f'_y = f'_z = \frac{1}{x + y + z}$$

and then we compute the second order partial derivatives and form the Hessian matrix

$$H(f) = -\frac{1}{(x + y + z)^2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

The principal minors of order one are all equal to $-1/(x + y + z)^2 < 0$, and all principal minors of higher order are zero. It follows that f is concave but not convex. The correct answer is alternative **C**.

Question 8.

The shaded region in the figure is closed and bounded, but not convex. The correct answer is alternative **B**.