BI NORWEGIAN BUSINESS SCHOOL

Solutions:	**GRA 60352**	**Mathematics**		
Examination date:	11.10.2013	15:00 – 16:00	Total no. of pages:	3
			No. of attachments:	0
Permitted examination support material:	A bilingual dictionary and BI-approved calculator TEXAS INSTRUMENTS BA II Plus			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Ordinary exam			Responsible department: Economics	

Correct answers: D-C-D-A-C-B-C-C

Question 1.

The linear system is consistent since it is homogeneous. It has $n - \text{rk}\, A = 4 - 2 = 2$ degrees of freedom. The correct answer is alternative **D**.

Question 2.

We form the matrix with the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ as columns, and compute its determinant

$$\begin{vmatrix} 0 & h-1 & h \\ h & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = h - 1$$

This shows that the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent when $h \neq 1$, and linearly dependent if $h = 1$. The correct answer is alternative **C**.

Question 3.

We reduce the matrix A to an echelon form:

$$\begin{pmatrix} 1 & 1 & -1 & -1 \\ 1 & 3 & h & -1 \\ 2 & 3 & 0 & h \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 2 & h+1 & 0 \\ 0 & 1 & 2 & h+2 \end{pmatrix} \dashrightarrow \begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 2 & h+2 \\ 0 & 0 & h-3 & -2h-4 \end{pmatrix}$$

There will be a pivot in the third row since all entries cannot be zero for any value of h. The correct answer is alternative **D**.

Question 4.

The characteristic equation of A is

$$\begin{vmatrix} 5-\lambda & 0 & -1 \\ 0 & 2-\lambda & 0 \\ 4 & 0 & -\lambda \end{vmatrix} = (2-\lambda)(\lambda^2 - 5\lambda + 4) = 0$$

Hence the eigenvalues of A are $\lambda = 2$, $\lambda = 1$ and $\lambda = 4$. The correct answer is alternative **A**.

Question 5.

We compute $A\mathbf{v}$ and compare with $\lambda\mathbf{v}$, and see that

$$\begin{pmatrix} 1 & s \\ 3 & -1 \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2+s \\ 5 \end{pmatrix}$$

is a multiple of $\mathbf{v}$ if and only if $s = 8$ (with $\lambda = 5$ in that case). The correct answer is alternative **C**.

Question 6.

The symmetric matrix of the quadratic form $f(x_1, x_2, x_3, x_4) = x_1^2 + 3x_1x_4 + 2x_2^2 + 6x_2x_4 + 3x_3^2 + 7x_4^2$ is given by

$$A = \begin{pmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \\ 3/2 & 3 & 0 & 7 \end{pmatrix}$$

The leading principal minors are $D_1 = 1$, $D_2 = 1 \cdot 2 = 2$, $D_3 = 1 \cdot 2 \cdot 3 = 6$, and $D_4 = |A|$ is given by

$$D_4 = \begin{vmatrix} 1 & 0 & 0 & 3/2 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \\ 3/2 & 3 & 0 & 7 \end{vmatrix} = 3(1(14-9) + 3/2(0-3)) = 3/2$$

Hence f is positive definite. The correct answer is alternative **B**.

Question 7.

We compute the first order derivatives to find stationary points, and find

$$3x^2 - 3y^2 - 3 = 0, \quad -6xy = 0, \quad -4z^3 + 4 = 0$$

This gives two stationary points $(\pm 1, 0, 1)$. We compute the Hessian matrix of f and find

$$H(f) = \begin{pmatrix} 6x & -6y & 0 \\ -6y & -6x & 0 \\ 0 & 0 & -12z^2 \end{pmatrix} = \begin{pmatrix} \pm 6 & 0 & 0 \\ 0 & \mp 6 & 0 \\ 0 & 0 & -12 \end{pmatrix}$$

Since $D_2 = -36$ at both stationary points, it follows that both are saddle points. The correct answer is alternative **C**.

Question 8.

The function $f(x, y, z) = x^2 + 4xz + 3y^2 - 2yz + 7z^2 + hx^4$ has Hessian matrix

$$H(f) = \begin{pmatrix} 2 + 12hx^2 & 0 & 4 \\ 0 & 6 & -2 \\ 4 & -2 & 14 \end{pmatrix}$$

Hence $D_1 = 2 + 12hx^2$, $D_2 = 6(2 + 12hx^2) = 12 + 72hx^2$ and $D_3 = 64 + 960hx^2$ since

$$|A| = (2 + 12hx^2)(84 - 4) + 4(0 - 24) = 64 + 960hx^2$$

We note that when a, b are constants with $a > 0$, then the sign of $a + bx^2$ is determined by the sign of b: If $b \geq 0$, then $a + bx^2 > 0$ for all x, and if $b < 0$ then $a + bx^2$ take both positive and negative values. It follows that $H(f)$ is positive definite for all (x, y, z) when $h \geq 0$, and that $H(f)$ is indefinite at some point (x, y, z) when $h < 0$. Hence f is concave for $h \geq 0$, and neither convex nor concave for $h < 0$. The correct answer is alternative **C**.