NORWEGIAN BUSINESS SCHOOL

Multiple-choice exam:	GRA 60352	Mathematics		
Examination date:	09.10.2015	15:00 – 16:00	Total no. of pages:	5 incl. attachments
			No. of attachments:	1 (1 page)
Permitted examination support material:	A bilingual dictionary and BI-approved calculator			
Answer sheets:	Answer sheet for multiple-choice examinations			
	Counts 20% of GRA 6035		The questions have equal weight	
Ordinary exam			Responsible department: Economics	

PLEASE READ THE FOLLOWING BEFORE YOU BEGIN!

- Students must themselves assure that the examination papers are complete.

- Students must provide the following information on the answer sheet:
    - Examination code
    - Personal initials
    - ID-nr

  The ID-nr must be recorded with both the appropriate numbers and by putting an "X" by the corresponding number in the columns below.

- Do not use pencils or pens with green ink when filling in answer sheets. Answer sheets must not be used for rough drafts.

- **All answers must be recorded with an "X" under the letter you believe corresponds with the correct answer.**

- **Cancel an "X" by filling in the box completely (boxes that are completely filled in will not be registered). "X" in two boxes for one question will be registered as a wrong answer.**

- The attached example shows you how the answer sheet would be filled in if A were the correct answer for question 1, B correct for question 2, C correct for question 3 and D correct for question 4. An "X" under E indicates that you choose not to answer question 5.

- **Your answers are to be recorded on the answer sheet. Answers written on the examination papers and not on the answer sheets will not be graded.**

- There is only <u>one</u> right answer for each question. Because the questions are weighted equally, it can be to your advantage to answer the easiest questions first.

- Wrong answers are given -1 point, unanswered questions get 0 points (indicated by an "X" next to E") and correct answers are given 3 points.

- You can keep the examination papers.

This exam has 8 questions

QUESTION 1.

Consider a homogeneous linear system $A \cdot \mathbf{x} = \mathbf{0}$, where A is a 3×5 matrix with $\operatorname{rk} A = 2$. **Which statement is true?**

    (a) The linear system has a unique solution
    (b) The linear system is inconsistent
    (c) The linear system has one degree of freedom
    (d) The linear system has more than one degree of freedom
    (e) I prefer not to answer.

QUESTION 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$
\mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ s \end{pmatrix}, \quad
\mathbf{v}_2 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad
\mathbf{v}_3 = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}
$$

Which statement is true?

    (a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all s
    (b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $s = 0$ or $s = 1$
    (c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $s \neq 1$
    (d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $s = 1$
    (e) I prefer not to answer.

QUESTION 3.

Compute the rank of the matrix

$$
A = \begin{pmatrix} 1 & 4 & -7 & 3 \\ 3 & 2 & 1 & 3 \\ 4 & 6 & t & 1-t \end{pmatrix}
$$

Which statement is true?

    (a) $\operatorname{rk} A = 2$ for all t
    (b) $\operatorname{rk} A = 3$ for $t \neq -6$ and $\operatorname{rk} A = 2$ for $t = -6$
    (c) $\operatorname{rk} A = 3$ for $t \neq -5$ and $\operatorname{rk} A = 2$ for $t = -5$
    (d) $\operatorname{rk} A = 3$ for all t
    (e) I prefer not to answer.

Consider the matrix

$$A = \begin{pmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{3} & 1 & 0 \\ 0 & 0 & -6 \end{pmatrix}$$

Which statement is true?

(a) A has three positive eigenvalues
(b) A has two positive and one negative eigenvalue
(c) A has one positive and two negative eigenvalues
(d) A has three negative eigenvalues
(e) I prefer not to answer.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & s & s \\ 0 & 2 & s \\ 0 & 0 & 3 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s = 1$
(c) A is diagonalizable exactly when $s \neq 1$
(d) A is diagonalizable exactly when $s = 0$
(e) I prefer not to answer.

A Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$ has transition matrix A given by

$$A = \begin{pmatrix} 0.74 & 0.13 \\ 0.26 & 0.87 \end{pmatrix}$$

and equilibrium state $\mathbf{x} = \left(\begin{smallmatrix} x \\ y \end{smallmatrix}\right)$. **Which statement is true?**

(a) $x = 1$ and $y = 2$
(b) $x = 0.74$ and $y = 0.26$
(c) $x = 13/74$ and $y = 61/74$
(d) $x = 1/3$ and $y = 2/3$
(e) I prefer not to answer.

Consider the quadratic form

$$f(x_1, x_2, x_3, x_4) = 2x_1^2 + 6x_1x_2 + 5x_2^2 - 2x_2x_3 + 3x_3^2 + 2x_3x_4 + 4x_4^2$$

Which statement is true?

(a) f is positive semidefinite but not positive definite
(b) f is positive definite
(c) f is indefinite
(d) f is negative definite
(e) I prefer not to answer.

For $a > 0$, consider the function $f(x, y) = x^a \sqrt{y}$ defined on $D_f = \{(x, y) : x > 0, y > 0\}$. **Which statement is true?**

    (a) f is a convex function for all a

    (b) f is a concave function for $a \leq 1/2$, and convex for $a > 1/2$

    (c) f is a concave function for $a \leq 1/2$, and neither convex nor concave for $a > 1/2$

    (d) f is a concave function for all a

    (e) I prefer not to answer.