This exam has 8 questions

QUESTION 1.

Consider a $5 \times 4$ linear system $Ax = b$ (five equations in four variables) such that the augmented matrix $(A|b)$ has rank 5. **Which statement is true?**

(a) The linear system has a unique solution  
(b) The linear system is inconsistent  
(c) The linear system has one degree of freedom  
(d) The linear system has more than one degree of freedom  
(e) I prefer not to answer.

QUESTION 2.

Consider the vectors $v_1, v_2, v_3$, given by

$$v_1 = \begin{pmatrix} 4 \\ t \\ 3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -4 \\ 3 \\ 1 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2t \\ 0 \\ 1 \end{pmatrix}$$

**Which statement is true?**

(a) The vectors $\{v_1, v_2, v_3\}$ are linearly independent for all $t$  
(b) The vectors $\{v_1, v_2, v_3\}$ are linearly dependent exactly when $t = 6$  
(c) The vectors $\{v_1, v_2, v_3\}$ are linearly dependent exactly when $t = 3$ or $t = 6$  
(d) The vectors $\{v_1, v_2, v_3\}$ are linearly dependent exactly when $t = 1$ or $t = 6$  
(e) I prefer not to answer.

QUESTION 3.

Compute the number of degrees of freedom of the linear system $A \cdot x = 0$ when

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 2 & 1 & t \\ 3 & 4 & t + 1 \\ 2 & -1 & 9 \end{pmatrix}$$

**Which statement is true?**

(a) There is one degree of freedom for all $t$  
(b) There is one degree of freedom when $t = 3$, and zero degrees of freedom when $t \neq 3$  
(c) There is one degree of freedom when $t = 7$, and zero degrees of freedom when $t \neq 7$  
(d) There are zero degrees of freedom for all $t$  
(e) I prefer not to answer.
Question 4.

Let $\lambda_1, \lambda_2, \lambda_3$ be the eigenvalues of the matrix

$$A = \begin{pmatrix} 17 & 0 & 9 \\ 0 & 6 & 0 \\ 16 & 0 & 17 \end{pmatrix}$$

Which statement is true?

(a) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1 \lambda_2 \lambda_3 > 0$
(b) $\lambda_1 + \lambda_2 + \lambda_3 > 0$ and $\lambda_1 \lambda_2 \lambda_3 < 0$
(c) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1 \lambda_2 \lambda_3 > 0$
(d) $\lambda_1 + \lambda_2 + \lambda_3 < 0$ and $\lambda_1 \lambda_2 \lambda_3 < 0$
(e) I prefer not to answer.

Question 5.

Consider the matrix $A$ given by

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & s & 1 \\ 0 & 0 & s - 1 \end{pmatrix}$$

Which statement is true?

(a) $A$ is diagonalizable for all $s$
(b) $A$ is diagonalizable exactly when $s = 1$
(c) $A$ is diagonalizable exactly when $s \neq 1, 2$
(d) $A$ is not diagonalizable for any $s$
(e) I prefer not to answer.

Question 6.

A Markov chain $x_{t+1} = Ax_t$ has transition matrix $A$ given by

$$A = \begin{pmatrix} 0.72 & 0.28 \\ 0.28 & 0.72 \end{pmatrix}$$

and equilibrium state $x = \begin{pmatrix} x \\ y \end{pmatrix}$. Which statement is true?

(a) $x = 28$ and $y = 72$
(b) $x = 1$ and $y = 1$
(c) $x = 28/72$ and $y = 44/72$
(d) $x = 1/2$ and $y = 1/2$
(e) I prefer not to answer.

Question 7.

Consider the quadratic form

$$f(x, y, z) = 3x^2 - 8xy - 4xz + 5y^2 - 4yz + 8z^2$$

Which statement is true?

(a) $f$ is positive semidefinite but not positive definite
(b) $f$ is positive definite but not positive semidefinite
(c) $f$ is indefinite
(d) $f$ is negative semidefinite
(e) I prefer not to answer.
Consider the function $f(x, y) = \sqrt{x^2 + y^2} + 3$. Which statement is true?

(a) $f$ is neither convex nor concave
(b) $f$ is convex but not concave
(c) $f$ is concave but not convex
(d) $f$ is both convex and concave
(e) I prefer not to answer.
**SVARARK TIL FLERVALGSEKSAMEN**

**ANSWER SHEET FOR MULTIPLE CHOICE EXAMINATION**

Eksamenskode: **GRA60352**  
Initialer: **NN**  
ID-nummer: *(SKAL fylles ut!)*  
ID-number: *(MUST be filled in!)*

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Write clearly!  
Record answer with **X**  
Cancel a cross with **-**  
Compl. filled boxes will not be registered

Dette svararket leses kun av en maskin. Ikke noe av det du skriver utenom de definerte feltene blir lest eller tatt hansyn til.

Ikke kloss på arket. Be heller om et nytt.

This answer sheet is only read by a machine. Answers or comments written on the examination paper or outside the boxes will not be graded.

Do not scribble on this sheet.

Please ask for a new answer sheet if you need one.

Examination

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