Exam	Midterm exam in GRA 6035 Mathematics
Date	October 13th, 2017 at 1500 - 1600

This exam has 8 questions

Question 1.

Consider a 4×6 linear system $A \cdot \mathbf{x} = \mathbf{b}$, where the coefficient matrix A has a pivot position in every row. **Which statement is true?**

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer.

Question 2.

Consider the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$, given by

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 3 \\ t \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent for all t
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly dependent exactly when $t = 7$
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $t = 7$
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ are linearly independent exactly when $t = 9$
(e) I prefer not to answer.

Question 3.

Consider the matrix

$$A = \begin{pmatrix} t & 1 & 1 \\ 1 & t & 1 \\ 1 & 1 & t \end{pmatrix}$$

Which statement is true?

(a) There is one value of t such that $\text{rk}(A) = 2$
(b) There are two values of t such that $\text{rk}(A) = 2$
(c) There are no values of t such that $\text{rk}(A) = 2$
(d) There are three values of t such that $\text{rk}(A) = 2$
(e) I prefer not to answer.

Question 4.

Consider the matrix

$$A = \begin{pmatrix} 3 & 5 & 2 \\ 0 & 2 & 0 \\ 1 & 3 & 4 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalues of multiplicity one, and no other eigenvalues
(e) I prefer not to answer.

Question 5.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & s & 1 \\ 0 & 1 & s \\ 0 & 0 & -1 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s = 1$
(c) A is diagonalizable exactly when $s = 0$
(d) A is not diagonalizable for any s
(e) I prefer not to answer.

Question 6.

Consider the quadratic form

$$f(x, y, z) = x^2 + 4xy + 2xz + 4y^2 + 4yz$$

Which statement is true?

(a) f is positive semidefinite but not positive definite
(b) f is positive definite
(c) f is negative definite
(d) f is indefinite
(e) I prefer not to answer.

Question 7.

Consider the function $f(x, y, z) = 1 - x^4 - 2x^2 + 4xz - y^2 - z^4 - 2z^2$. **Which statement is true?**

(a) f has a saddle point at $(x, y, z) = (0, 0, 0)$.
(b) f has a global maximum point at $(x, y, z) = (0, 0, 0)$.
(c) f has a global minimum point at $(x, y, z) = (0, 0, 0)$.
(d) f has a local maximum point at $(x, y, z) = (0, 0, 0)$, but no global maximum value.
(e) I prefer not to answer.

Question 8.

Two firms compete in a market. Initially, Firm A has 80% of the market, leaving Firm B with 20%. We model quarter-to-quarter changes in market shares as a Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$, where

$$\mathbf{x}_{t+1} = \begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} 0.80 & 0.20 \\ 0.20 & 0.80 \end{pmatrix} \cdot \begin{pmatrix} x_t \\ y_t \end{pmatrix} = A \cdot \mathbf{x}_t$$

and where x_t and y_t are the market shares of Firm A and Firm B after t quarters. In the long run, the equilibrium market shares are given by

$$\mathbf{x} = \begin{pmatrix} x \\ y \end{pmatrix} = \lim_{t\to\infty} \mathbf{x}_t = \lim_{t\to\infty} \begin{pmatrix} x_t \\ y_t \end{pmatrix}$$

Which statement is true?

(a) Equilibrium market share x of Firm A is at least 75%.
(b) Equilibrium market share x of Firm A is between 50% and 75%.
(c) Equilibrium market share x of Firm A is 50%.
(d) Equilibrium market share x of Firm A is less than 50%.
(e) I prefer not to answer.