Exam	Midterm exam in GRA 6035 Mathematics
Date	October 12th, 2018 at 1500 - 1600

This exam has 8 questions

Question 1.

Consider a 5×6 linear system $A \cdot \mathbf{x} = \mathbf{b}$, where $\text{rk}(A) = 5$. **Which statement is true?**

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer.

Question 2.

Consider the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ given by

$$\mathbf{v}_1 = \begin{pmatrix} t \\ 2 \\ 3 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 6 \\ t \\ 9+t \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly independent for all t
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly dependent when $t = 1$
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly dependent when $t = 3$
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly dependent when $t = 6$
(e) I prefer not to answer.

Question 3.

Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & -1 & 4 \\ 2 & 4 & 0 & 6 \\ t & -1 & 5 & 3 \end{pmatrix}$$

Which statement is true?

(a) For all values of t, we have that $\text{rk}(A) = 3$
(b) There is one value of t such that $\text{rk}(A) = 2$, otherwise $\text{rk}(A) = 3$
(c) There is one value of t such that $\text{rk}(A) = 3$, otherwise $\text{rk}(A) = 2$
(d) For all values of t, we have that $\text{rk}(A) = 2$
(e) I prefer not to answer.

Question 4.

Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalues of multiplicity one, and no other eigenvalues
(e) I prefer not to answer.

Question 5.

Consider the matrix A given by

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & s & 0 \\ 1 & 0 & 3 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s \neq 2$ and $s \neq 4$
(c) A is diagonalizable exactly when $s \neq 2$
(d) A is diagonalizable exactly when $s \neq 4$
(e) I prefer not to answer.

Question 6.

A Markov chain $\mathbf{x}_{t+1} = A \cdot \mathbf{x}_t$ has transition matrix A and equilibrium state $\mathbf{v}$ given by

$$A = \begin{pmatrix} 0.40 & 0.20 & 0.10 \\ 0.40 & 0.60 & 0.10 \\ 0.20 & 0.20 & 0.80 \end{pmatrix}, \qquad \mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

Which statement is true?

(a) $v_2 < 0.20$
(b) $0.20 < v_2 < 0.25$
(c) $0.25 < v_2 < 0.30$
(d) $0.30 < v_2$
(e) I prefer not to answer.

Question 7.

Consider the quadratic form

$$f(x, y, z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$$

Which statement is true?

(a) f is positive semi-definite but not positive definite
(b) f is positive definite
(c) f is negative definite
(d) f is indefinite
(e) I prefer not to answer.

Question 8.

Consider the function $f(x, y, z) = 1 - (x - y + z)^4$. **Which statement is true?**

(a) The point $(x, y, z) = (1, 1, 0)$ is not a local maximum point for f
(b) The point $(x, y, z) = (1, 1, 0)$ is a local maximum point for f, but not a global maximum
(c) The point $(x, y, z) = (1, 1, 0)$ is a global maximum point for f, but f is not concave
(d) The point $(x, y, z) = (1, 1, 0)$ is a global maximum point for f, and f is concave
(e) I prefer not to answer.