Exam	Midterm exam in GRA 6035 Mathematics
Date	January 07th, 2019 at 0900 - 1000

This exam has 8 questions

Question 1.
A 4×6 linear system $A \cdot \mathbf{x} = \mathbf{b}$ has 3 degrees of freedom. **Which statement is true?**

(a) $\text{rk}(A) = 4$
(b) $\text{rk}(A) = 3$
(c) $\text{rk}(A) = 2$
(d) $\text{rk}(A) = 1$
(e) I prefer not to answer.

Question 2.
Consider the vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ given by

$$\mathbf{v}_1 = \begin{pmatrix} 2 \\ t \\ 3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 3 \\ 6 \\ t \end{pmatrix}$$

Which statement is true?

(a) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly independent for all t
(b) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly dependent for all t
(c) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly dependent when $t = 4$, and linearly independent otherwise
(d) The vectors $\{\mathbf{v}_1, \mathbf{v}_2\}$ are linearly independent when $t = 4$, and linearly dependent otherwise
(e) I prefer not to answer.

Question 3.
Consider the matrix

$$A = \begin{pmatrix} 1 & 3 & -1 & 4 \\ 1 & 1 & 1 & 2 \\ t & -1 & 5 & 3 \end{pmatrix}$$

Which statement is true?

(a) For all values of t, we have that $\text{rk}(A) = 3$
(b) There is one value of t such that $\text{rk}(A) = 2$, otherwise $\text{rk}(A) = 3$
(c) There is one value of t such that $\text{rk}(A) = 3$, otherwise $\text{rk}(A) = 2$
(d) For all values of t, we have that $\text{rk}(A) = 2$
(e) I prefer not to answer.

Question 4.
Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 2 \\ 0 & 2 & 3 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalues of multiplicity one, and no other eigenvalues
(e) I prefer not to answer.

Question 5.
Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & 1 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all s
(b) A is diagonalizable exactly when $s \neq 1$
(c) A is not diagonalizable for any value of s
(d) A is diagonalizable exactly when $s = 0$
(e) I prefer not to answer.

Question 6.
A 3×4 linear system $A \cdot \mathbf{x} = \mathbf{b}$ has infinitely many solutions and 1 degree of freedom. **Which statement is true?**

(a) $\dim \mathrm{Null}(A) = 1$
(b) $\dim \mathrm{Null}(A) = 2$
(c) $\dim \mathrm{Null}(A) = 3$
(d) $\dim \mathrm{Null}(A) = 0$
(e) I prefer not to answer.

Question 7.
Consider the quadratic form

$$f(x, y, z, w) = 5x^2 + 4xy + y^2 + 3z^2 + 2zw + w^2$$

Which statement is true?

(a) f is positive semi-definite but not positive definite
(b) f is positive definite
(c) f is negative definite
(d) f is indefinite
(e) I prefer not to answer.

Question 8.
Consider the function $f(x, y, z) = x^3 + y^3 + z^3 - 3(x + y + z)$. **Which statement is true?**

(a) f has a local maximum point, but not a local minimum point
(b) f has a local minimum point, but not a local maximum point
(c) f has a local maximum point and a local minimum point, but no saddle points
(d) f has a local maximum point, a local minimum point, and one or more saddle points
(e) I prefer not to answer.