Exam	Midterm exam in GRA 6035 Mathematics
Date	October 08th, 2021 at 1500 - 1600

Question 1.
Consider the linear system with augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 7 \\ 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 1 & 0 & 4 \end{array}\right)$$

Which statement is true?

(A) The linear system is inconsistent
(B) The linear system has a unique solution
(C) The linear system has one degree of freedom
(D) The linear system has two degrees of freedom
(E) I prefer not to answer

Question 2.
Let the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ and $\mathbf{v}_4$ be the column vectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 0 & 3 \\ 5 & 4 & 6 & 1 \end{pmatrix}$$

Which statement is true?

(A) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a base of $\text{Col}(A)$
(B) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a base of $\text{Col}(A)$
(C) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of $\text{Col}(A)$
(D) $\{\mathbf{v}_1, \mathbf{v}_2\}$ is a base of $\text{Col}(A)$
(E) I prefer not to answer

Question 3.
Consider the matrix

$$A = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 5 \end{pmatrix}$$

Which statement is true?

(A) A has three distinct eigenvalues
(B) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(C) A has an eigenvalue of multiplicity three
(D) A has one eigenvalues of multiplicity one, and no other eigenvalues
(E) I prefer not to answer

Question 4.
Consider the matrix

$$A = \begin{pmatrix} t+1 & t & t-1 \\ t & t & t \end{pmatrix}$$

Which statement is true?

(A) For $t = 0$, we have that $\text{rk}(A) = 1$, otherwise $\text{rk}(A) = 2$
(B) For $t = 0$ and $t = 1$, we have that $\text{rk}(A) = 1$, otherwise $\text{rk}(A) = 2$
(C) For $t = 0$, $t = 1$ and $t = -1$, we have that $\text{rk}(A) = 1$, otherwise $\text{rk}(A) = 2$
(D) For all values of t, we have that $\text{rk}(A) = 2$
(E) I prefer not to answer

Question 5.

Consider the matrix A given by

$$A = \begin{pmatrix} 3 & 0 & s \\ 0 & s & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

Which statement is true?

(A) A is diagonalizable if and only if $s \neq 1$
(B) A is diagonalizable if and only if $s \neq 1$ and $s \neq 3$
(C) A is diagonalizable if and only if $s \neq 3$
(D) A is diagonalizable for all values of s
(E) I prefer not to answer.

Question 6.

Consider the quadratic form

$$f(x, y, z, w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 6z^2$$

Which statement is true?

(A) f is indefinite
(B) f is negative semi-definite but not negative definite
(C) f is positive semi-definite but not positive definite
(D) f is positive definite
(E) I prefer not to answer

Question 7.

Consider the function

$$f(x, y, z) = x^4 + y^4 + z^4 - 4xyz$$

Which statement is true?

(A) The point $(x, y, z) = (1, 1, 1)$ is not a stationary point of f
(B) The point $(x, y, z) = (1, 1, 1)$ is a local minimum point of f
(C) The point $(x, y, z) = (1, 1, 1)$ is a local maximum point of f
(D) The point $(x, y, z) = (1, 1, 1)$ is a saddle point of f
(E) I prefer not to answer

Question 8.

Let A be a 4×4 matrix, such that $\mathcal{B} = \{\mathbf{v}_1\}$ is a base for the eigenspace E_0 of A with

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$$

Which statement is true?

(A) $\text{rk}(A) = 1$
(B) $\text{rk}(A) = 2$
(C) $\text{rk}(A) = 3$
(D) $\text{rk}(A) = 4$
(E) I prefer not to answer