Exam	Midterm exam in GRA 6035 Mathematics
Date	January 27th, 2022 at 1700 - 1800

Question 1.
Consider the linear system with augmented matrix

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 4 \\ 2 & -1 & 3 & 0 & 1 \\ 3 & 0 & 4 & 1 & 4 \end{array}\right)$$

Which statement is true?

(a) The linear system is inconsistent
(b) The linear system has a unique solution
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer

Question 2.
We consider the four column vectors of the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 0 & 3 \\ 5 & 4 & 6 & a \end{pmatrix}$$

Which statement is true?

(a) The vectors are linearly independent for all values of a
(b) The vectors are linearly independent if and only if $a = 6$
(c) The vectors are linearly dependent if and only if $a = 6$
(d) The vectors are linearly dependent for all values of a
(e) I prefer not to answer

Question 3.
Consider the matrix

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalue of multiplicity one, and no other eigenvalues
(e) I prefer not to answer

Question 4.
A Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$ has transition matrix A given by

$$A = \begin{pmatrix} 0.56 & 0.22 \\ 0.44 & 0.78 \end{pmatrix}$$

and equilibrium state $\mathbf{x} = (x, y)$. **Which statement is true?**

(a) $y < 0.25$
(b) $0.25 \leq y < 0.50$
(c) $0.50 \leq y < 0.75$
(d) $y \geq 0.75$
(e) I prefer not to answer.

Question 5.
Consider the matrix A given by

$$A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & s & 2 \\ 1 & 0 & 2 \end{pmatrix}$$

Which statement is true?

(a) A is not diagonalizable for any value of s
(b) A is diagonalizable if and only if $s \neq 1$ and $s \neq 3$
(c) A is diagonalizable if and only if $s \neq 3$
(d) A is diagonalizable for all values of s
(e) I prefer not to answer.

Question 6.
Consider the quadratic form

$$f(x, y, z, w) = 3x^2 + 2xy + 8xz - 2xw + y^2 + 4yz + 2yw + 7z^2 + 4w^2$$

Which statement is true?

(a) f is positive definite
(b) f is positive semi-definite but not positive definite
(c) f is negative semi-definite but not negative definite
(d) f is indefinite
(e) I prefer not to answer

Question 7.
Consider the function

$$f(x, y, z) = 3x^2 + 2xy + 8xz + y^2 + 4yz + 7z^2$$

Which statement is true?

(a) f does not have any stationary points
(b) f has a saddle point
(c) f has a local minimum but not a global minimum
(d) f has a global minimum
(e) I prefer not to answer

Question 8.
Let A be a symmetric 3×3 matrix such that $|A| = -6$ and $\text{tr}(A) = 4$. **Which statement is true?**

(a) A is positive definite
(b) A is negative definite
(c) A is indefinite
(d) It is not possible to determine the definiteness of A based on the given information
(e) I prefer not to answer