Question 1.
Consider the linear system with augmented matrix
$$\left(\begin{array}{cccc|c} 1 & 0 & 7 & 1 & 13 \\ 3 & 2 & 0 & 2 & 3 \\ 7 & 4 & 7 & 5 & 19 \end{array} \right)$$
Which statement is true?

    (a) The linear system is inconsistent
    (b) The linear system has a unique solution
    (c) The linear system has one degree of freedom
    (d) The linear system has two degrees of freedom
    (e) I prefer not to answer

Question 2.
We consider the three column vectors of the matrix
$$A = \begin{pmatrix} 1 & 2 & 5 \\ 1 & 1 & 4 \\ 2 & 0 & 6 \\ 1 & 3 & a \end{pmatrix}$$
Which statement is true?

    (a) The vectors are linearly independent for all values of a
    (b) The vectors are linearly independent if and only if $a = 6$
    (c) The vectors are linearly dependent if and only if $a = 6$
    (d) The vectors are linearly dependent for all values of a
    (e) I prefer not to answer

Question 3.
Consider the matrix
$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$
Which statement is true?

    (a) A has three distinct eigenvalues
    (b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
    (c) A has an eigenvalue of multiplicity three
    (d) A has one eigenvalue of multiplicity one, and no other eigenvalues
    (e) I prefer not to answer

Question 4.
Consider the quadratic form $f(x, y, z) = 3x^2 + 8xy + 2xz + 6y^2 + 4yz$. **Which statement is true?**

    (a) f is positive definite
    (b) f is positive semi-definite but not positive definite
    (c) f is negative semi-definite but not negative definite
    (d) f is indefinite
    (e) I prefer not to answer

Question 5.

A Markov chain $\mathbf{x}_{t+1} = A\mathbf{x}_t$ has transition matrix A given by

$$A = \begin{pmatrix} 0.50 & 0.30 & 0.00 \\ 0.00 & 0.70 & 0.50 \\ 0.50 & 0.00 & 0.50 \end{pmatrix}$$

Which statement is true?

    (a) There is an equilibrium state (x, y, z) with $y < 0.25$

    (b) There is an equilibrium state (x, y, z) with $0.25 \leq y < 0.50$

    (c) There is an equilibrium state (x, y, z) with $0.50 \leq y$

    (d) The Markov chain is not regular

    (e) I prefer not to answer.

Question 6.

Consider the function $f(x, y, z) = 3x^2 + 8xy + 2xz + 6y^2 + 4yz + z^2$. **Which statement is true?**

    (a) f does not have any stationary points

    (b) f has a saddle point

    (c) f has a local minimum but not a global minimum

    (d) f has a global minimum

    (e) I prefer not to answer

Question 7.

Consider the matrix A given by

$$A = \begin{pmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & a & -7 & -2 \\ 3 & 10 & -7 & 16 & 7 \end{pmatrix}$$

Which statement is true?

    (a) $\operatorname{rk} A = 3$

    (b) $\operatorname{rk} A = 3$ when $a = 9$, otherwise $\operatorname{rk} A = 4$

    (c) $\operatorname{rk} A = 2$ when $a = 9$, otherwise $\operatorname{rk} A = 4$

    (d) $\operatorname{rk} A = 4$

    (e) I prefer not to answer.

Question 8.

Let A be a 4×4 matrix with characteristic equation $\lambda^4 - 3\lambda^2 = 0$ such that A is not diagonalizable.
Which statement is true?

    (a) $\operatorname{rk} A = 1$

    (b) $\operatorname{rk} A = 2$

    (c) $\operatorname{rk} A = 3$

    (d) It is impossible to determine $\operatorname{rk} A$ from the given information.

    (e) I prefer not to answer