Exam	Mock Midterm exam in GRA 6035 Mathematics
Date	October 9th, 2020 at 1300 - 1400

Question 1.

Consider the linear system with augmented matrix

$$\left(\begin{array}{cccc|c} 0 & 0 & 3 & 1 & 4 \\ 0 & 0 & 1 & 5 & 7 \\ 1 & 1 & 0 & 5 & 13 \\ 4 & 0 & 0 & 1 & 5 \end{array}\right)$$

Which statement is true?

(a) The linear system has a unique solution
(b) The linear system is inconsistent
(c) The linear system has one degree of freedom
(d) The linear system has two degrees of freedom
(e) I prefer not to answer.

Question 2.

Consider the matrix

$$A = \begin{pmatrix} 1 & t & -t \\ 5 & -t & t \\ 4 & 2 & 0 \end{pmatrix}$$

Which statement is true?

(a) $\operatorname{rk} A = 3$ for all values of t
(b) $\operatorname{rk} A = 2$ for a unique value of t, otherwise $\operatorname{rk} A = 3$
(c) $\operatorname{rk} A = 1$ for a unique value of t, otherwise $\operatorname{rk} A = 3$
(d) $\operatorname{rk} A < 3$ for two different values of t
(e) I prefer not to answer.

Question 3.

Let $V = \operatorname{Col}(A)$ and let $\mathbf{v}_i$ be the i'th column vector of A for $i = 1, 2, 3, 4$ when

$$A = \begin{pmatrix} 3 & 1 & 7 & 0 \\ 2 & 0 & 5 & 8 \\ 3 & 5 & 5 & 2 \end{pmatrix}$$

Which statement is true?

(a) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is a base of V
(b) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ are linearly independent, but is not a base of V
(c) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a base of V, and $\mathbf{v}_4$ is a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$.
(d) $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of V, and $\mathbf{v}_3$ is a linear combination of $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$.
(e) I prefer not to answer.

Question 4.

Consider the quadratic form $f(x, y, z) = x^2 + 4xy - 2xz + 5y^2 - 4yz + z^2$. **Which statement is true?**

(a) f is indefinite
(b) f is negative semi-definite but not negative definite
(c) f is positive semi-definite but not positive definite
(d) f is positive definite
(e) I prefer not to answer.

Question 5.
Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 4 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

Which statement is true?

(a) A has three distinct eigenvalues
(b) A has an eigenvalue of multiplicity two, and another eigenvalue of multiplicity one
(c) A has an eigenvalue of multiplicity three
(d) A has one eigenvalues of multiplicity one, and no other eigenvalues
(e) I prefer not to answer.

Question 6.
Consider the function $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$. **Which statement is true?**

(a) The function f has both a local minimum and a local maximum point.
(b) The function f has a local minimum but no local maximum point.
(c) The function f has a local maximum but no local minimum point.
(d) All stationary points of f are saddle points.
(e) I prefer not to answer.

Question 7.
Consider the function $f(x, y, z) = x^2 + 4xy - 2xz + 5y^2 - 4yz + hz^2 + z^4$ with parameter h. **Which statement is true?**

(a) f is convex for all values of h
(b) f is convex for $h \geq 0$
(c) f is convex for $h \geq 1$
(d) f is not convex for any value of h
(e) I prefer not to answer.

Question 8.
Consider the matrix

$$A = \begin{pmatrix} 0 & 0 & 2 \\ 4 & 0 & 0 \\ 0 & s^3 & 0 \end{pmatrix}$$

Which statement is true?

(a) A is diagonalizable for all values of s
(b) A is diagonalizable for $s = 0$, but not for $s \neq 0$
(c) A is diagonalizable for $s \neq 0$, but not for $s = 0$
(d) A is not diagonalizable for any value of s
(e) I prefer not to answer.