Key Problems

Problem 1.
Solve the linear systems with the following augmented matrices:

a) $\left(\begin{array}{ccc|c} 1 & 3 & 4 & 11 \\ 5 & 1 & 8 & 15 \\ 4 & 5 & 9 & 23 \end{array}\right)$ b) $\left(\begin{array}{ccc|c} 4 & 5 & 11 & 23 \\ 2 & -1 & 3 & 3 \\ 3 & 2 & 7 & 12 \end{array}\right)$ c) $\left(\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & 4 & 8 \\ 1 & 3 & 1 & 5 & 18 & 28 \\ 2 & 4 & 2 & 9 & 31 & 48 \end{array}\right)$

Problem 2.
Find the rank of the following matrix:

$$A = \begin{pmatrix} 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 3 & 1 \\ 3 & 0 & 2 & 3 & 1 \\ 3 & 6 & 0 & 0 & -1 \\ 0 & 0 & 2 & 0 & -1 \end{pmatrix}$$

Problem 3.
We consider the following linear system. Find all solutions that satisfies $x + w = y + z$:

$$\begin{array}{rcrcrcrcr} x & + & y & + & 2z & + & 4w & = & 6 \\ x & + & 2y & + & 4z & - & 2w & = & 9 \\ x & + & 3y & + & 9z & + & 7w & = & 24 \end{array}$$

Problem 4.
We consider the homogeneous linear system with the following coefficient matrix:

$$A = \begin{pmatrix} 6 & 6 & 3 & 6 \\ 5 & 5 & -1 & 4 \\ 8 & 7 & 7 & 5 \end{pmatrix}$$

Describe the set of solutions geometrically. How many degrees of freedom are there? Does this change if we change the red coefficient in the second row?

Exercise problems

Problems from the textbook: [E] 1.1 - 1.17

Answers to Key Problems

Problem 1.

a) (1,2,1) b) No solutions c) $(2-s,2-t,s,4-3t,t)$

Problem 2.

5

Problem 3.

$(1,-9/5,3,1/5)$

Problem 4.

The linear system has one degree of freedom, and the solutions form a straight line in $\mathbb{R}^4$. This does not change when we change the red coefficient.