Key Problems

Problem 1.

Find the equilibrium states and determine their stability. Sketch the solution curve y = y(t).

a)
$$y' = 6 - 2y$$

b)
$$y' = y^2 - 4$$

c)
$$y' = 5y(1 - y/10)$$

Problem 2.

Solve the differential equations:

a)
$$y'' + 6y' - 16y = 16t - 22$$

b)
$$y'' + 6y' + 9y = 4e^{-t}$$

c)
$$y'' - 3y' + 2y = 3e^{2t}$$

d)
$$y'' - y = t^2$$

Problem 3.

Show that there are separable, non-linear differential equations with the following functions as particular solutions:

a)
$$y = \sqrt{t^2 - 3}$$

b)
$$y = \frac{2}{1 - t^2}$$

Exercise Problems

Textbook problems: [E] 7.24 - 7.34

Exam problems [Final 12/2015] Q2, [Final 11/2017] Q2, [Final 11/2018] Q2ac, Q5,

[Final 01/2021] Q3bc

Answers to Key Problems

Problem 1.

a) $y_e = 3$ is globally asymptotically stable

b) $y_e=-2$ is stable (but not globally asymptotically stable), $y_e=2$ is unstable

c) $y_e = 0$ is unstable, $y_e = 10$ is stable (but not globally asymptotically stable)

Problem 2.

a)
$$y = C_1 e^{-8t} + C_2 e^{2t} + 1 - t$$

b)
$$y = C_1 e^{-3t} + C_2 t e^{-3t} + e^{-t}$$

c)
$$y = C_1 e^{2t} + C_2 e^t + 3t e^{2t}$$

d)
$$y = C_1 e^t + C_2 e^{-t} - t^2 - 2$$