Key Problems

Problem 1.

Write the systems of differential equations on matrix form and solve them:

Problem 2.

Solve the systems of differential equations:

a)
$$\mathbf{y}' = \begin{pmatrix} -5 & 0 & 1\\ 0 & -3 & 0\\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} 1\\ 0\\ 0 \end{pmatrix}$$
 b) $\mathbf{y}' = \begin{pmatrix} 2 & 1 & 1\\ -1 & 2 & 0\\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}, \quad \mathbf{y}(0) = \begin{pmatrix} -1\\ -3\\ 8 \end{pmatrix}$

Problem 3.

Solve the differential equation y''' + 4y'' + y' - 6y = 0.

Problem 4.

Let $y(t) = 3e^{-2t} - 5e^t + 12e^{-3t}$.

- a) Find a linear second order differential equation that has y as a particular solution.
- b) Find a linear third order differential equation that has y as a particular solution.
- c) Find a 3×3 matrix A such that $\mathbf{y}' = A\mathbf{y}$ has $\mathbf{y} = (y, y', y'')$ as a particular solution.

Problem 5.

Find the equilibrium states and determine their stability:

a) y'' + 5y' + 6y = 0b) y'' + 5y' + 6y = 12c) y'' + y' - 20y = 1d) y''' + 4y'' + y' - 6y = 0

Exercise Problems

Problems from the textbook	[E] 9.1 - 9.7
Final exam problems	11/2018 Q2,Q5, 01/2019 Q2, 01/2020 Q3, 03/2021 Q3bd

Answers to Key Problems

Problem 1.

a)
$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{7t} - C_2 e^{-3t} \\ C_1 e^{7t} + C_2 e^{-3t} \end{pmatrix}$$
 b) $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} C_1 e^{4t} - C_2 e^{-t} \\ 4C_1 e^{4t} + C_2 e^{-t} \end{pmatrix}$ c) $\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 4C_1 e^{2t} - C_2 e^{-3t} + 1 \\ C_1 e^{2t} + C_2 e^{-3t} - 1 \end{pmatrix}$

Problem 2.

a)
$$\mathbf{y} = \frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-4t} - \frac{1}{2} \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot e^{-6t}$$
 b) $\mathbf{y} = \begin{pmatrix} -2 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} e^{2t} + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} e^{3t}$

Problem 3. $y = C_1 \cdot e^t + C_2 \cdot e^{-2t} + C_3 e^{-3t}$

Problem 4.

More than one solution is possible:

a)
$$y'' + y' - 2y = 48e^{-3t}$$
 b) $y''' + 4y'' + y' - 6y = 0$ c) $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -1 & -4 \end{pmatrix}$

Problem 5.

a) $\mathbf{y}_e = (0,0)$ is globally asymptotically stable

c) $\mathbf{y}_e = (-1/20,0)$ is unstable

b) $\mathbf{y}_e = (2,0)$ is globally asymptotically stable d) $\mathbf{y}_e = (0,0,0)$ is unstable