## Key Problems

**Problem 1.**
Solve the difference equations:

a) $y_{t+1} - 6y_t = 10t + 3$

b) $y_{t+2} - 5y_{t+1} + 6y_t = 2t$

c) $y_{t+2} - 4y_{t+1} + 4y_t = 1$

d) $y_{t+2} + y_{t+1} - 2y_t = 6$

**Problem 2.**
Write the systems of difference equations on matrix form and solve them:

a) $\begin{array}{rcl} y_{t+1} & = & 2y_t + 5z_t \\ z_{t+1} & = & 5y_t + 2z_t \end{array}$

b) $\begin{array}{rcl} y_{t+1} & = & z_t \\ z_{t+1} & = & 4y_t + 3z_t \end{array}$

c) $\begin{array}{rcl} y_{t+1} & = & y_t + 4z_t + 3 \\ z_{t+1} & = & y_t - 2z_t - 3 \end{array}$

**Problem 3.**

a) Solve the difference equation $x_{t+1} = 3x_t + 4,\ x_0 = 2$ and compute $x_5$.

b) You borrow an amount $K$. The interest rate per period is $r$. The repayment is 500 in the first period, and increases with 10 for each subsequent period. Show that the outstanding balance $b_t$ after period $t$ satisfies the difference equation
$$b_{t+1} = (1+r)b_t - (500 + 10t), \quad b_0 = K$$
and solve this difference equation.

**Problem 4.**
We consider a model for housing prices, where $p_t$ is the price after $t$ years. The model is given by the difference equation
$$p_{t+2} - 2p_{t+1} + p_t = -15, \quad p_0 = 695,\ p_1 = 743$$

a) Solve the difference equation.

b) We define $d_t = p_{t+1} - p_t$ to be the change in housing prices. Show that $d_{t+1} - d_t$ is constant, and use this to determine when housing prices will increase and when housing prices will decrease.

**Problem 5.**
Solve the systems of difference equations:

a) $\mathbf{y}_{t+1} = \begin{pmatrix} -5 & 0 & 1 \\ 0 & -3 & 0 \\ 1 & 0 & -5 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

b) $\mathbf{y}_{t+1} = \begin{pmatrix} 2 & 1 & 1 \\ -1 & 2 & 0 \\ 3 & -1 & 1 \end{pmatrix} \cdot \mathbf{y}_t, \quad \mathbf{y}(0) = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$

## Exercise Problems

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|---|---|
| Problems from the textbook | [E] 8.1 - 8.9, 9.8 |
| Final exam problems | Final exam 11/2019 Q5, 01/2021 Q3a, 03/2021 Q3a |

# Answers to Key Problems

**Problem 1.**

a) $y_t = C \cdot 6^t - 2t - 1$

b) $y_t = C_1 \cdot 2^t + C_2 \cdot 3^t + t + 3/2$

c) $y_t = (C_1 + C_2 t) \cdot 2^t + 1$

d) $y_t = C_1 + C_2 \cdot 2^t + 2t$

**Problem 2.**

a) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} C_1\, 7^t - C_2\,(-3)^t \\ C_1\, 7^t + C_2\,(-3)^t \end{pmatrix}$

b) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} C_1\, 4^t - C_2\,(-1)^t \\ 4C_1\, 4^t + C_2\,(-1)^t \end{pmatrix}$

c) $\begin{pmatrix} y_t \\ z_t \end{pmatrix} = \begin{pmatrix} 4C_1\, 2^t - C_2\,(-3)^t + 3/4 \\ C_1\, 2^t + C_2\,(-3)^t - 3/4 \end{pmatrix}$

**Problem 3.**

a) $x_t = 4 \cdot 3^t - 2,\ x_5 = 970$

b) $b_t = \left(K - \frac{10}{r^2} - \frac{500}{r}\right)(1+r)^t + \frac{10}{r}t + \frac{10}{r^2} + \frac{500}{r}$

**Problem 4.**

a) $p_t = 695 + 55.5t - 7.5t^2$

b) $d_{t+1} - d_t = -15$, $d_t > 0$ for $t = 0, 1, 2, 3$ and that $d_t < 0$ for $t \geq 4$

**Problem 5.**

a) $\mathbf{y}_t = \frac{1}{2}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot (-4)^t - \frac{1}{2}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot (-6)^t$

b) $\mathbf{y}_t = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \cdot 2^t + \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cdot 3^t$