Key Problems

Problem 1.
We consider the vectors $\mathbf{v}_1 = (1,3,4),\ \mathbf{v}_2 = (-1,3,4),\ \mathbf{v}_3 = (5,3,4),\ \mathbf{v}_4 = (6,4,5),\ \mathbf{v}_5 = (4,2,3)$.

a) Is $\mathbf{v}_3$ in $\operatorname{span}(\mathbf{v}_1, \mathbf{v}_2)$?
b) Express $\mathbf{v}_5$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ if possible.
c) Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$ linearly independent vectors?
d) Are $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_5$ linearly independent vectors?
e) Determine the dimension of $V = \operatorname{span}(\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5)$, and find a base of V.
f) Express $\mathbf{v}_5$ as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4$ in a different way than in (b), if possible.

Problem 2.
Find a parametric description of the line through the points (1,2,1) and (4,5,3) in $\mathbb{R}^3$. Determine the intersection points (x,y,z) of this line and the plane $x - y + z = 6$.

Problem 3.
Determine $\dim V$ and $\dim W$ when $V = \operatorname{Col}(A)$, $W = \operatorname{Null}(A)$, and A is the 3×5 matrix A given below, and find a base of V and W:

$$A = \begin{pmatrix} 1 & -1 & 5 & 6 & 4 \\ 2 & 4 & -2 & -2 & -2 \\ 3 & 5 & -1 & -1 & -1 \end{pmatrix}$$

Problem 4.
Let A be a 8×8 matrix with rank given by $\operatorname{rk}(A) = 7$ and let $\mathbf{b}$ be a vector in $\mathbb{R}^8$. Determine:

a) $\dim \operatorname{Null}(A)$ and $\dim \operatorname{Col}(A)$
b) The number of solutions of $A\mathbf{x} = \mathbf{0}$
c) The number of solutions of $A\mathbf{x} = \mathbf{b}$
d) The number of solutions of $A\mathbf{x} = \mathbf{0}$ that satisfies $x_1 + x_2 + \cdots + x_8 = 1$

Exercise problems

Problems from the textbook:	[E] 2.1 - 2.16
Exam problems:	[Midterm 10/2019] Question 1, 2, 8
	[Midterm 10/2022] Question 1, 2, 7

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Answers to Key Problems

Problem 1.

a) Yes

b) $\mathbf{v}_5 = 6\mathbf{v}_1 - 4\mathbf{v}_2 - \mathbf{v}_4$

c) Yes

d) Yes

e) $\dim V = 3$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of V

f) $\mathbf{v}_5 = 2\mathbf{v}_3 - \mathbf{v}_4$

Problem 2.

Parametric description: $(x,y,z) = (1+3t, 2+3t, 1+2t)$. Intersection point: $(x,y,z) = (10,11,7)$.

Problem 3.

a) $\dim V = 3$, and $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4\}$ is a base of V when $\mathbf{v}_i$ is the i'th column vectors of A

b) $\dim W = 2$, and $\{\mathbf{w}_1, \mathbf{w}_2\}$ is a base for W when $\mathbf{w}_1 = (-3,2,1,0,0)$, $\mathbf{w}_2 = (-6,4,0,1,1)$

Problem 4.

a) $\dim \text{Null}(A) = 1$ and $\dim \text{Col}(A) = 7$

b) Infinitely many solutions (one degree of freedom)

c) Infinitely many solutions (one degree of freedom) if $\mathbf{b}$ is a linear combination of the columns of A, otherwise no solutions

d) No solutions if $(1,1,\dots,1)$ is a linear combination of the rows of A, otherwise one unique solution.