

Key Problems

Problem 1.

Determine the definiteness of the symmetric matrix:

$$\text{a) } A = \begin{pmatrix} 7 & 4 \\ 4 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} -1 & 1 \\ 1 & -3 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 2 & 3 & -5 \\ 3 & 7 & 0 \\ -5 & 0 & 35 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} -1 & -2 & -2 \\ -2 & -4 & -4 \\ -2 & -4 & -2 \end{pmatrix}$$

Problem 2.

Find the symmetric matrix of the quadratic form, and determine its definiteness:

$$\text{a) } f(x, y) = x^2 - 8xy + 3y^2$$

$$\text{b) } f(x, y, z) = 2x^2 - 2xz + 3y^2 + z^2$$

$$\text{c) } f(x, y, z) = 3x^2 + 4xy - 4xz + 3y^2 + 4yz + 8z^2$$

$$\text{d) } f(x, y, z, w) = xw - yz$$

Problem 3.

Determine the definiteness of the symmetric matrix:

$$A = \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{pmatrix}$$

Problem 4.

Find an orthogonal matrix P such that $P^T A P = D$ is a diagonal matrix:

$$\text{a) } A = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 4 & 0 \\ 1 & 0 & 4 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

Problem 5.

Find the equilibrium state $\mathbf{v}$ of the Markov chains with transition matrix A . Is the Markov chain regular?

$$\text{a) } A = \begin{pmatrix} 0.30 & 0.15 \\ 0.70 & 0.85 \end{pmatrix}$$

$$\text{b) } A = \begin{pmatrix} 0.86 & 0.42 \\ 0.14 & 0.58 \end{pmatrix}$$

$$\text{c) } A = \begin{pmatrix} 0.75 & 0.02 & 0.10 \\ 0.20 & 0.90 & 0.20 \\ 0.05 & 0.08 & 0.70 \end{pmatrix}$$

$$\text{d) } A = \begin{pmatrix} 1 & 0.3 \\ 0 & 0.7 \end{pmatrix}$$

$$\text{e) } A = \begin{pmatrix} 0.2 & 0.4 & 0 \\ 0.8 & 0.4 & 0.7 \\ 0 & 0.2 & 0.3 \end{pmatrix}$$

Exercise Problems

Problems from the textbook: [E] 4.8 - 4.17

Exam problems:

Midterm exam 10/2018 Question 1-7

Midterm exam 01/2019 Question 1-7

Exercise problems for Midterm exam Question 1-20

Answers to Key Problems

Problem 1.

- a) Positive definite
- b) Negative definite
- c) Positive definite
- d) Positive semi-definite
- e) Indefinite

Problem 2.

- a) Indefinite
- b) Positive definite
- c) Positive semi-definite
- d) Indefinite

Problem 3.

Positive semi-definite

Problem 4.

a) $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$

b) $P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}$

c) $P = \frac{1}{\sqrt{6}} \begin{pmatrix} -\sqrt{3} & -1 & \sqrt{2} \\ 0 & 2 & \sqrt{2} \\ \sqrt{3} & -1 & \sqrt{2} \end{pmatrix}$

Problem 5.

a) $\mathbf{v} = \begin{pmatrix} 3/17 \\ 14/17 \end{pmatrix}$ (regular)

b) $\mathbf{v} = \begin{pmatrix} 3/4 \\ 1/4 \end{pmatrix}$ (regular)

c) $\mathbf{v} = \begin{pmatrix} 2/15 \\ 10/15 \\ 3/15 \end{pmatrix}$ (regular)

d) $\mathbf{v} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ (not regular)

e) $\mathbf{v} = \begin{pmatrix} 7/25 \\ 14/25 \\ 4/25 \end{pmatrix}$ (regular)