Linear approximation of a function

Let $f(x_1,\dots,x_n)$ be a C^1 function in a neighbourhood at the point $(x_1^*,\dots,x_n^*)$. Then the linear approximation of f is

$$f(x_1,\dots,x_n) \simeq f(x_1^*,\dots,x_n^*) + \frac{\partial f}{\partial x_1}(x_1^*,\dots,x_n^*)\cdot(x_1-x_1^*) + \frac{\partial f}{\partial x_2}(x_1^*,\dots,x_n^*)\cdot(x_2-x_2^*) + \dots + \frac{\partial f}{\partial x_n}\cdot(x_1^*,\dots,x_n^*)\cdot(x_n-x_n^*)$$

Ex: $f(x,y) = e^{xy}$ at $(0,0)$ and $(1,1)$

$f(0,0) = \underline{1}$

$f'_x = e^{xy}\cdot y \quad f'_x(0,0) = \underline{0}$

$f'_y = e^{xy}\cdot x \quad f'_y(0,0) = \underline{0}$

When (x,y) close to $(0,0)$:

$$f(x,y) \cong 1 + 0\cdot(x-0) + 0\cdot(y-0) = 1$$

$f(1,1) = e \quad f'_x(1,1) = e \quad f'_y(1,1) = e$

When (x,y) close to $(1,1)$:

$$f(x,y) \simeq e + e\cdot(x-1) + e\cdot(y-1) = ex + ey - e$$