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FORELESNING

MET1180
MATEMATIKK

Eivind Eriksen

DES 5, 2016

OPPSUMMERING HØST -16

Plan:

1. Oppsummering av pensum
   - Finansmatematikk 1
   - Likninger og ulikheter 2
   - Funksjoner og grafer 3
   - Derivasjon 4

2. Gjennomgang Flervalgseksamen MET11802 11/2015

A Finansmatematikk

1. Årsslutt 2003: $500000 \cdot \left(1 + \frac{0{,}0325}{2}\right) = 508.125$

$$508.125 \cdot 1{,}0325^x = 1.000.000$$

$$1{,}0325^x = 1.000.000 / 508.125$$

$$\ln 1{,}0325^x = \ln(1{,}968)$$

$$x \cdot \ln 1{,}0325 = \ln(1{,}968)$$

$$x = \frac{\ln(1{,}968)}{\ln(1{,}0325)} \simeq 21{,}17 \to 22 \text{ år}$$

2025 D

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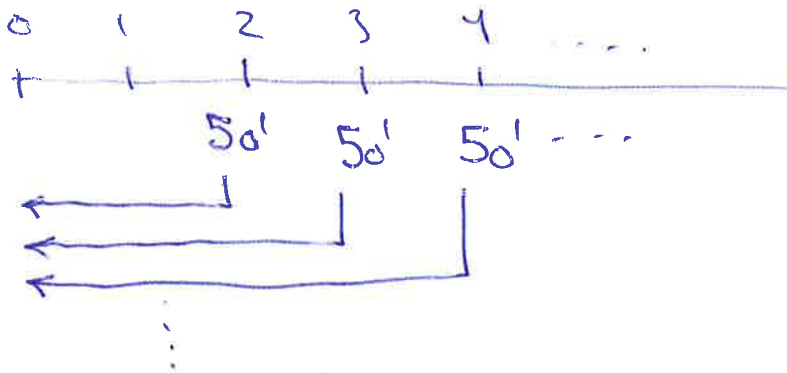
2.



$r = 8\% = 0{,}08$

Nåverdi $\dfrac{50\cdot000}{1{,}08^2} + \dfrac{50000}{1{,}08^3} + \dfrac{50000}{1{,}08^4} + \ldots$

geom. $k = 1/1{,}08$

$$= a_1 \cdot \frac{1}{1-k} = \frac{50000}{1{,}08^2} \cdot \frac{1}{1-\frac{1}{1{,}08}}$$

$$= \frac{50000}{1{,}08^2 - 1{,}08} = \frac{50\cdot000}{1{,}08\,(0{,}08)}$$

$$\approx 578\cdot704$$

C

Viktig formel:

Geometrisk rekke

a_1 første ledd
k kvotient
n ledd

$$S_n = a_1 \cdot \frac{k^n - 1}{k-1} = a_1 \cdot \frac{1-k^n}{1-k}$$

a_1 første ledd
k kvotient
uendelig rekke

$$S = a_1 \cdot \frac{1}{1-k} \quad \text{hvis } -1 < k < 1$$

divergent ellers

Formel for etterskuddsannuitet:

Fast beløp A per år, rente r, uendelig kontantstrøm, første betaling etter et år

Nåverdi: $\dfrac{A}{r}$

Justert:

$$\frac{A}{r} \cdot \frac{1}{1+r} = \frac{A}{r(1+r)}$$

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3. $S(x) = 3 + x + x^2/3 + \dots$

Geometrisk rekke:

$a_1 = 3$

$k = \frac{a_2}{a_1} = \frac{x}{3}$

$\frac{a_3}{a_2} = \frac{x^2/3}{x} = \frac{x}{3}$

konvergent: $-1 < k = \frac{x}{3} < 1$

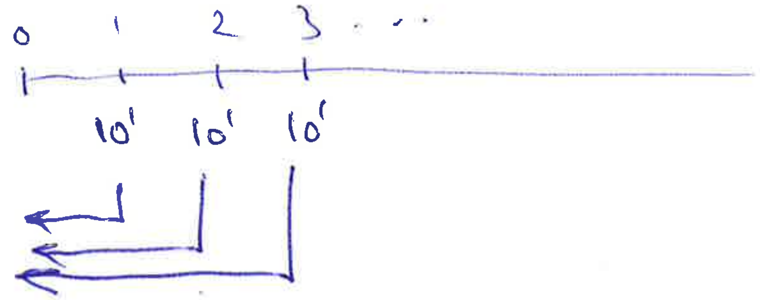
$-1 < \frac{x}{3}$ og $\frac{x}{3} < 1$

$-3 < x$ og $x < 3$ dvs $-3 < x < 3$

$x = 2$: $S(2) = a_1 \cdot \frac{1}{1-k} = 3 \cdot \frac{1}{1-2/3} = \frac{3 \cdot 3}{1/3 \cdot 3} = 9$

c

4.



r = diskonteringsrente (kont.)

nåverdi: $\frac{10.000}{e^{r \cdot 1}} + \frac{10.000}{e^{r \cdot 2}} + \frac{10.000}{e^{3r}} + \dots = 60.000$

geom. uendelig rekke

$a_1 = \frac{10000}{e^r}$

$k = \frac{1}{e^r}$

$\frac{10.000}{e^r} \cdot \frac{1}{1 - 1/e^r} = 60.000$

$\frac{10.000}{e^r - 1} = 60000 \quad | \cdot (e^r - 1)$

$\frac{10.000}{60.000} = \frac{\cancel{60.000}}{\cancel{60.000}} \cdot (e^r - 1)$

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$$e^r - 1 = \frac{1}{6}$$

$$e^r = \frac{1}{6} + 1 = \frac{7}{6}$$

$$r = \ln(7/6) \simeq 0,154 \simeq \underline{15,4\%}$$

D

Formel: Etterskuddsannuitet A, kont. diskonteringsrente r, første betaling etter 1 år, i det uendelige $\Big\}$ $\dfrac{A}{e^r - 1}$

B Likninger og ulikheter

5. $$\frac{x-8}{x^2-3x-4} \geq 1$$

Flytter alle ledd over på VS, og setter opp fortegnsdiagram

$$\frac{x-8}{x^2-3x-4} - 1 \geq 0$$

$$\frac{x-8}{x^2-3x-4} - \frac{(x^2-3x-4)}{x^2-3x-4} \geq 0$$

$$\frac{-x^2+4x-4}{x^2-3x-4} \geq 0$$

$$\frac{-(x-2)^2}{(x-4)(x+1)} \geq 0$$

$$-x^2+4x-4=0$$

$$x = \frac{-4 \pm \sqrt{16-16}}{-2} = \underline{2}$$

$$x^2-3x-4=0$$

$$x = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2}$$

$$= 4, -1$$

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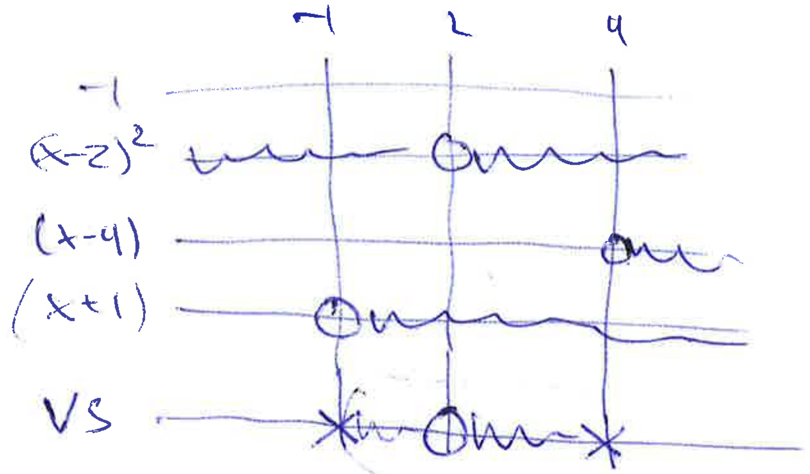
$$\frac{-(x-2)^2}{(x-4)(x+1)} \geq 0$$



Løsn: $-1 < x < 4$ (A)

$(-1, 4)$ ←

Intervaller

$(\ \ ) = < \ \ >$ åpne

(endepkt er ikke med)

$[\ \ ]$ lukket

(endepkt er med)

6. $x^3 - 7x = 6$

$x^3 - 7x - 6 = 0$

$x=1$: $-12 \neq 0$

$x=-1$: $0 = 0$ løsn.

$(x+1)(x^2 - x - 6) = 0$

$x = -1$ eller $x^2 - x - 6 = 0$

$$x = \frac{1 \pm \sqrt{1+24}}{2}$$

$$= \frac{1 \pm 5}{2} = 3, -2$$

(D)

Mulige heltallsløsn: Går opp i -6

$\pm 1, \pm 2, \pm 3, \pm 6$

$(x^3 - 7x - 6) : (x+1) = x^2 - x - 6$

$-(x^3 + x^2)$

$-x^2 - 7x - 6$

$-(-x^2 - x)$

$-6x - 6$

$-6x - 6$

0

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7. $1-\sqrt{x} = \sqrt{x-5} \quad | (\ )^2$

$(1-\sqrt{x})^2 = x-5$

$1-2\sqrt{x}+\cancel{x} = \cancel{x}-5$

$\frac{6}{2} = \frac{2\sqrt{x}}{2}$

$3 = \sqrt{x} \quad | (\ )^2$

$9 = x$

$\underline{x=9}$

Setter prøve:

VS: $1-\sqrt{9} = -2$

HS: $\sqrt{9-5} = 2$

$x=9$ ikke løsn.

⇓

Ingen løsn. A

8. $x^3-ax^2-3x+1=0$

$x=2$ er løsn.: $2^3 - a\cdot 2^2 - 3\cdot 2 + 1 = 0$

$8-4a-6+1=0$

$3-4a=0$

$a = \underline{3/4}$

$x^3 - \frac{3}{4}x^2 - 3x + 1 = 0$

$$\boxed{(x-2)\cdot\left(x^2+\frac{5}{4}x-\frac{1}{2}\right) = 0}$$

$x^3 - \frac{3}{4}x^2 - 3x + 1 : x-2 = \underline{x^2 + \frac{5}{4}x - \frac{1}{2}}$

$-(x^3-2x^2)$

$\frac{5}{4}x^2 - 3x + 1$

$-(\frac{5}{4}x^2 - \frac{5}{2}x)$

$-\frac{1}{2}x+1$

$-\frac{1}{2}x+1$

0

$x^2+\frac{5}{4}x-\frac{1}{2}=0$

$x = \frac{-5/4 \pm \sqrt{\frac{25}{16}+2}}{2}$

$x_1 = \frac{-5/4 + \sqrt{\frac{25}{16}+2}}{2}$

$x_2 = \frac{-5/4 - \sqrt{\frac{25}{16}+2}}{2}$

$x_1+x_2 = -\frac{5}{4}$ C

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Els: $x^6 - 7x^3 + 6 = 0$

$u = x^3$ $\quad u^2 - 7u + 6 = 0$

$u = 6$ eller $u = 1$

$x^3 = 6 \qquad x^3 = 1$

$x = \sqrt[3]{6} \qquad x = 1$

C Funksjoner og grafer

skrå/hor. as.

9. $f(x) = \dfrac{x^3 - 3x^2 + x - 1}{x^2 - 4x + 3} = \boxed{x+1} + \dfrac{2x-4}{x^2-4x+3} \to 0$

Vertikale asymptoter:

$x^2 - 4x + 3 = 0$

$x = 3,\ x = 1$ vertikale as. $a = 2$

$3^3 - 3 \cdot 3^2 + 3 - 1 = 2 \neq 0$

$1^3 - 3 \cdot 1^2 + 1 - 1 \quad -2 \neq 0$

Skrå/horisontale asympt.

$x^3 - 3x^2 + x - 1 : x^2 - 4x + 3 = x + 1$

$-(x^3 - 4x^2 + 3x)$

$x^2 - 2x - 1$

$-(x^2 - 4x + 3)$

$2x - 4$

$y = x + 1 \qquad b = 1$

$ab = 2$ B

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Grafer:

Rette linjer:

$$\begin{cases} y = ax + b & \begin{cases} a: \text{stign.tall} \\ b: \text{skj. med } y\text{-aksen} \end{cases} \\ x = c & (\text{vertikale}) \end{cases}$$

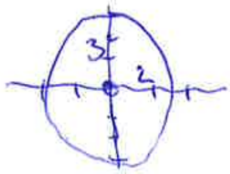
Sirkler: $(x-x_0)^2 + (y-y_0)^2 = r^2$ $\begin{cases} \text{Sentrum i } (x_0, y_0) \\ r > 0 \text{ radius} \end{cases}$

Ellipse: $\dfrac{(x-x_0)^2}{a^2} + \dfrac{(y-y_0)^2}{b^2} = 1$

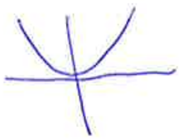
$a, b > 0$ halvakser

(x_0, y_0) sentrum

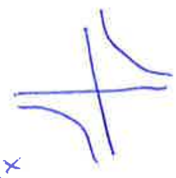
Eks: $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$

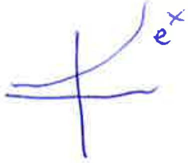


Parabel: $y = x^2$



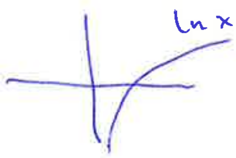
Hyperbel: $y = 1/x$





Eksponential: $y = e^x$

Logaritme: $y = \ln x$



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D Derivasjon.

10. $f = \frac{x^4+1}{(x^2+1)(x+1)} = \frac{u}{v}$ $f'(0)$ = stigningstallet til tangente i $x=0$

$f' = \frac{u'v - uv'}{v^2} = \frac{4x^3\cdot(x^2+1)(x+1) - (x^4+1)\cdot\left(2x\cdot(x+1) + (x^2+1)\cdot 1\right)}{(x^2+1)^2(x+1)^2}$

x^3+x^2+x+1

$= \frac{4x^3(x^2+1)(x+1) - (x^4+1)(2x(x+1)+(x^2+1)\cdot 1)}{(x^2+1)^2(x+1)^2}$

$f'(0) = \frac{-1(0+1)}{1^2\cdot 1^2} = \underline{\underline{-1}}$ B

11. $El_p\, D(p) = \frac{p}{D(p)} \cdot D'(p)$

$= \frac{p\cdot(-9)}{240-9p} = -1$

$\boxed{El_p\, D(p) = \frac{p}{D(p)}\cdot D'(p)}$

$\frac{-9p}{240-9p} = -1 \quad \Big| \cdot (240-9p)$

$-9p = -1\cdot(240-9p)$

$-9p = -240 + 9p$

$\frac{240}{18} = \frac{18p}{18}$

$p = \frac{240}{18} = \underline{\frac{40}{3}}$ C

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12.

$$f(x) = \frac{\sqrt{x}}{x-2} = \frac{u}{v}, \quad x \geq 0, \; x \neq 2$$

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

$$= \frac{\frac{1}{2\sqrt{x}} \cdot (x-2) - \sqrt{x} \cdot 1}{(x-2)^2} \quad \begin{matrix} \cdot 2\sqrt{x} \\ \cdot 2\sqrt{x} \end{matrix}$$

$$= \frac{(x-2) - 2x}{2\sqrt{x} \cdot (x-2)^2}$$

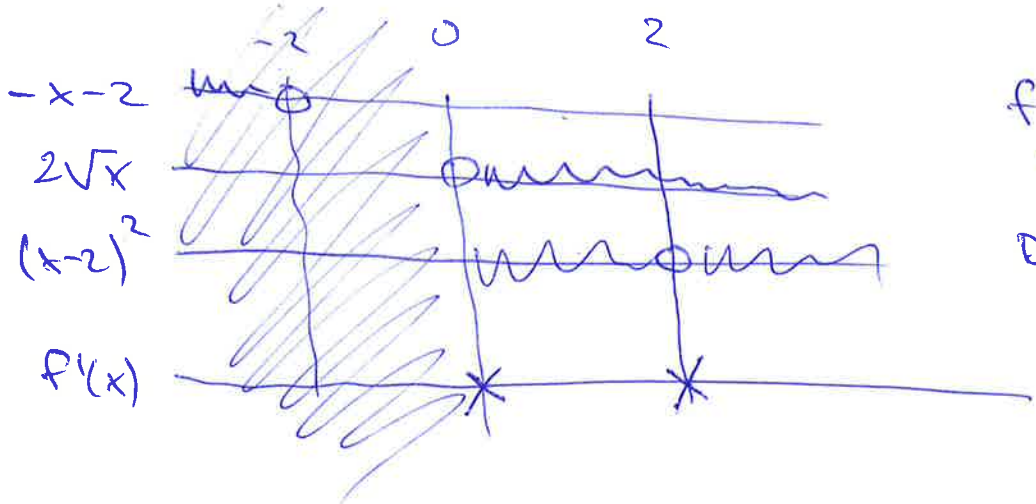
$$= \frac{-x-2}{2\sqrt{x} \cdot (x-2)^2}$$

f voksende i $[a,b]$
$\Updownarrow$
$f'(x) \geq 0$ for alle x i (a,b)

Derivasjonsregler:

$(u \cdot v)' = u' \cdot v + u \cdot v'$

$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u v'}{v^2}$



f er aldri voksende (D)

$D_f = [0,2) \cup (2, \rightarrow)$

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13. $f(x) = \ln(x^2+1) - \ln(x-1)$, $x > 1$

$$f'(x) = \frac{1}{x^2+1}\cdot 2x - \frac{1}{x-1}\cdot 1$$

$$= \frac{2x}{x^2+1} - \frac{1}{x-1}$$

$$= \frac{2x(x-1) - 1\cdot(x^2+1)}{(x^2+1)(x-1)}$$

$$= \frac{2x^2-2x-x^2-1}{(x^2+1)(x-1)}$$

$$= \frac{x^2-2x-1}{(x^2+1)(x-1)} = \frac{(x-x_1)(x-x_2)}{(x^2+1)(x-1)}$$

Kjerneregelen

$f = f(u)$, $u = u(x)$

$\Downarrow$

$f'(x) = f'(u)\cdot u'(x)$

$$\frac{df}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$$

$f(x) = \ln(x^2+1) = \ln(u)$, $u = x^2+1$

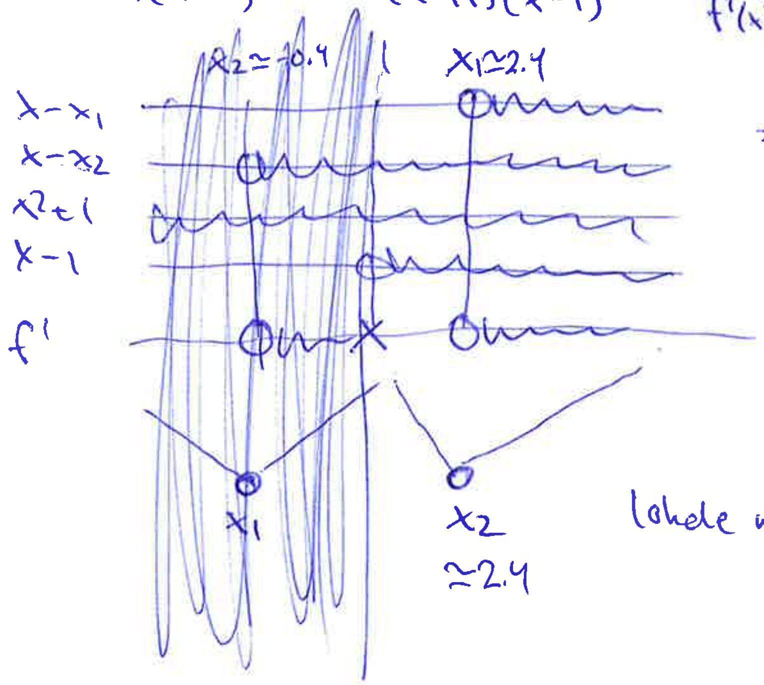
$$f'(x) = \frac{1}{u}\cdot 2x = \frac{1}{x^2+1}\cdot 2x$$

$x^2-2x-1=0$

$$x = \frac{2\pm\sqrt{4+4}}{2} = \frac{2\pm\sqrt{8}}{2} = 1\pm\frac{\sqrt{8}}{2}$$

$x_1 = 1+\sqrt{8}/2 \simeq 2{,}4$

$x_2 = 1-\sqrt{8}/2 \simeq -0{,}4$



lokale min = globalt min

$$\lim_{x\to\infty} \ln(x^2+1) - \ln(x-1) = \infty$$

$$= \lim_{x\to\infty} = \ln\left|\frac{x^2+1}{x-1}\right|$$

ingen nullps.

D

$f(2{,}4) = \ln|2.4^2+1| - \ln 1{,}4 > 0$

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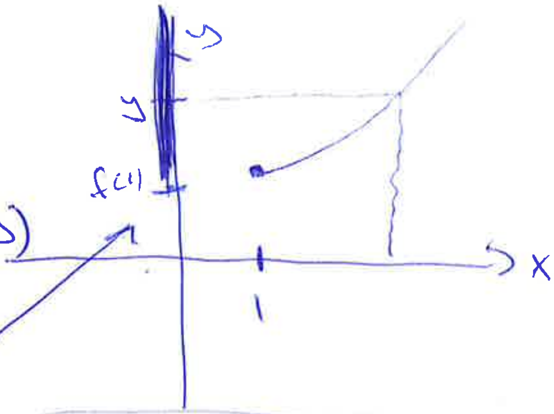
14. $f(x) = (x^2+2x+3)\,e^x$, $D_f = [1, \to)$

$$f'(x) = (2x+2)\cdot e^x + (x^2+2x+3)\cdot e^x$$

$$= e^x \cdot (2x+2+x^2+2x+3)$$

$$= \underline{e^x (x^2+4x+5)}$$



$x^2+4x+5=0$

$x = \dfrac{-4 \pm \sqrt{16-4\cdot 5}}{2}$

ingen løsn.

e^x

x^2+4x+5

f'

f voksende i D_f

$\Downarrow$

f har omvendt funksjon

f avtagende i D_f

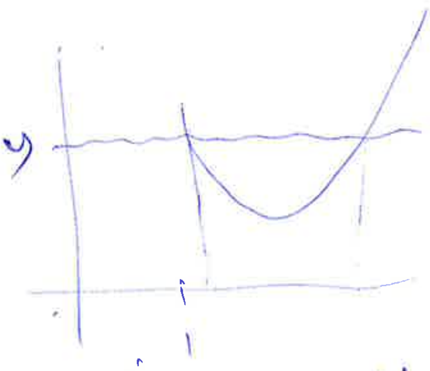
$\Downarrow$

f har omvendt funksjon

$\underline{f^{-1} \text{ finnes}}$, $\boxed{D_{f^{-1}} = V_f}$

$$D_{f^{-1}} = V_f = [f(1), \to)$$

$$= [6e, \to)$$ C



ingen omvendt funksjon

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15.

$$\lim_{x\to 1} \frac{\ln x - x + 1}{x^2 - 2x + 1} = \lim_{x\to 1} \frac{\frac{1}{x} - 1 + 0}{2x - 2}$$

"$\frac{0}{0}$"

L'Hopitals regel

$$= \lim_{x\to 1} \frac{\frac{1}{x} - 1}{2x-2} = \lim_{x\to 1} \frac{-1\cdot x^{-2} - 0}{2} = \lim_{x\to 1} \frac{-\frac{1}{x^2}}{2} = -\frac{1}{2}$$

"$\frac{0}{0}$"

D

Husk:
$f''(x)$
vendepkt
konveks/konkav