NORWEGIAN BUSINESS SCHOOL
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FORELESNING 10

MET 1180
MATEMATIKK

EIVIND ERIKSEN

OCT 26, 2016

Plan:

① Logaritmer
② Derivasjon (hvis tid) ← Neste gang
③ Gjesteforelesning: Finansminister Siv Jensen kl 0930 – 10.15

Pensum:
[EJ 3.13, 4.1

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① Logaritmer:

Defn: $\log_a(x)$ er den omvendte funksjonen til $f(x) = a^x$

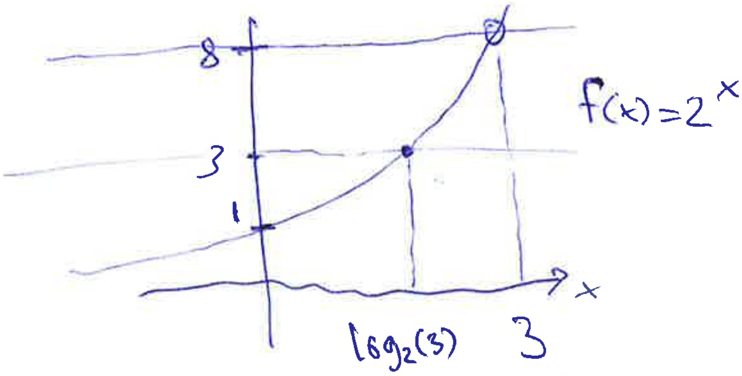
logaritme-funksjonen med grunntall a

eksponentialfunksjonen med grunntall a

$a > 0,\ a \neq 1$

Eks: $\log_2(8) = 3$

$\log_2(3)$



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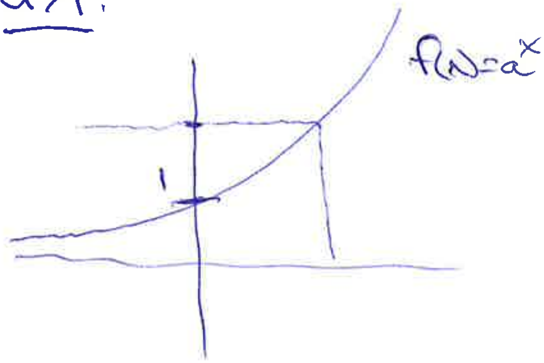
$a>1$:

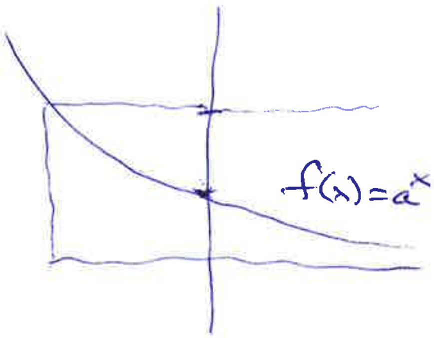


- f er voksende / avtagende
- $D_f = \mathbb{R}$
- $V_f = (0, \rightarrow)$

$\Downarrow$

$g(x) = \log_a(x)$ er veldefinert (den omvendte funksjonen eksisterer)

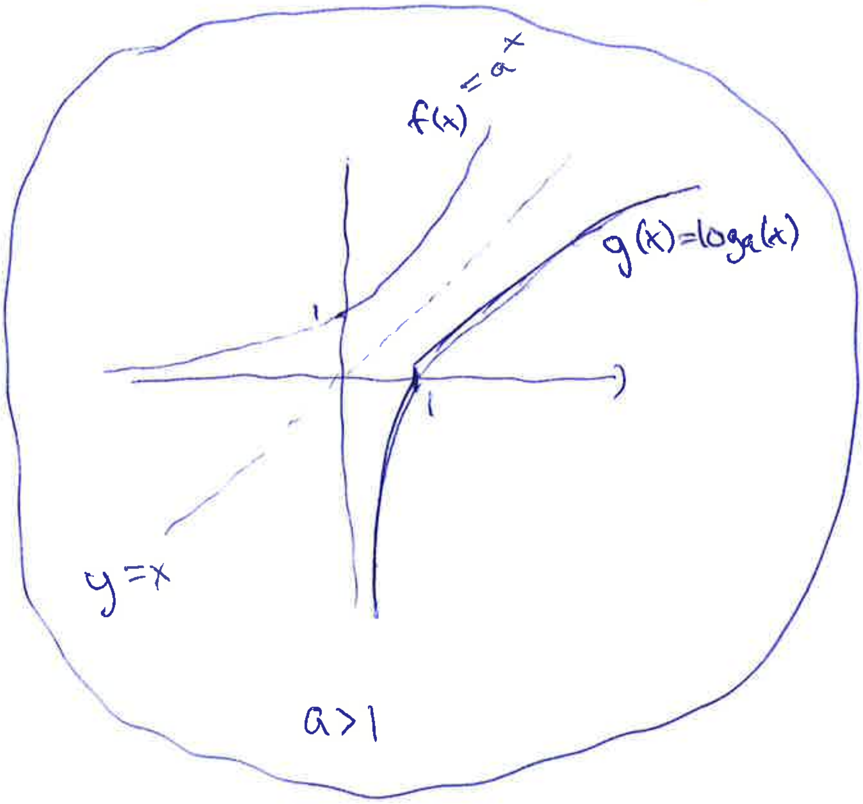
$0 < a < 1$:

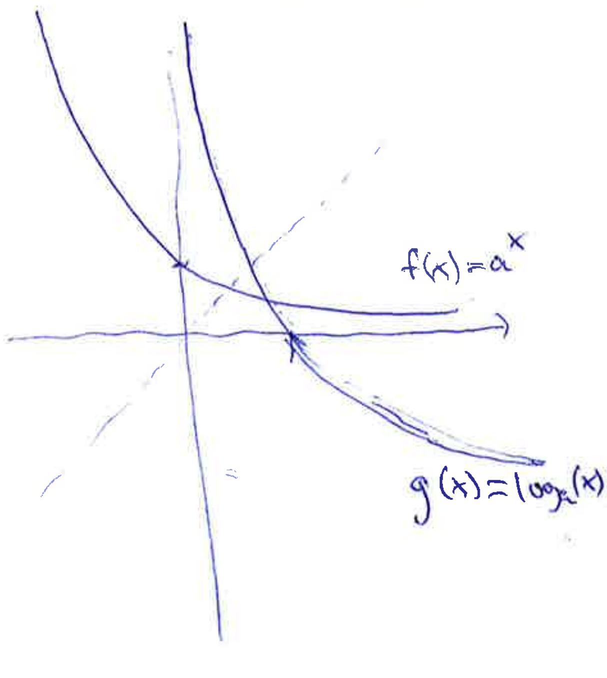


$D_g = (0, \rightarrow)$

$V_g = \mathbb{R}$

Konklusjon: For $a>0$, $a \neq 1$, er $g(x) = \log_a(x)$ er kontinuerlig funksjon med $D_g = \{x : x > 0\} = (0, \rightarrow)$





$0<a<1$

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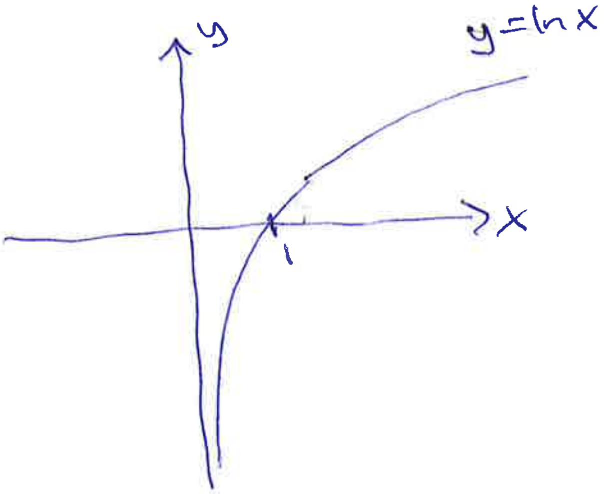
Viktige spesialtilfeller:

① $a = e = 2{,}71828\ldots$: $\quad \log_a(x) = \ln(x)$

logaritmefunksjon ved grunntall $e = 2{,}71828\ldots$

naturlig logaritme

② $a = 10$: $\quad \log_{10}(x) = \log(x)$



Regneregler for logaritmer

i) $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

ii) $\log_a(x/y) = \log_a(x) - \log_a(y)$

iii) $\log_a(x^n) = n \cdot \log_a(x)$

Gjelder for alle grunntall a

potenser:

$a^{x+y} = a^x \cdot a^y$

$a^{x-y} = a^x / a^y$

$a^{xy} = (a^x)^y$

$\ln(xy) = \ln(x) + \ln(y)$

$\ln(x/y) = \ln(x) - \ln(y)$

$\ln(x^n) = n \cdot \ln(x)$

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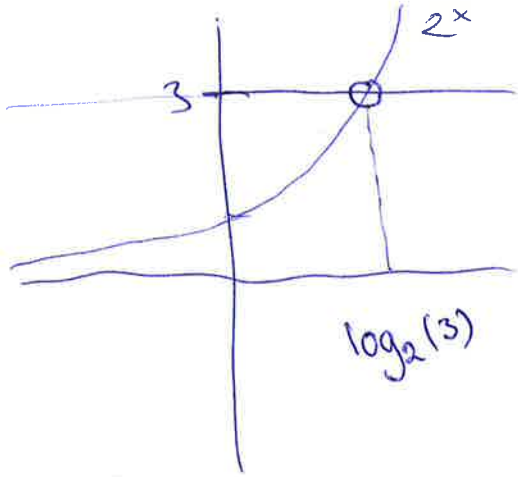
Eks:

$$2^x = 3$$

$$\log_2(2^x) = \log_2(3)$$

$$x = \log_2(3)$$

2^x og $\log_2(x)$ er omvendte funksjoner



$$2^x = 3$$

$$\ln(2^x) = \ln(3)$$

$$\frac{x \cdot \cancel{\ln(2)}}{\cancel{\ln(2)}} = \frac{\ln(3)}{\ln(2)}$$

$$x = \frac{\ln 3}{\ln 2} \simeq 1{,}58$$

$$x = \log_2(3)$$

På kalkulator

$\boxed{\ln(x)}$

Eks:

$$a^x = y$$

$$\log_a(a^x) = \log_a(y)$$

$$x = \log_a(y)$$

$$\ln(a^x) = \ln(y)$$

$$x \cdot \ln(a) = \ln(y)$$

$$x = \frac{\ln(y)}{\ln(a)}$$

Regneregel for omskriving av logaritmer:

$$\log_a(x) = \frac{\ln(x)}{\ln(a)}$$

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Løsning av likninger med logaritmer

$$2\cdot 3^x = 10$$
$$\ln(2\cdot 3^x) = \ln(10)$$
$$\ln(2) + x\cdot\ln(3) = \ln(10)$$
$$x\,\ln(3) = \ln(10) - \ln(2)$$
$$x = \frac{\ln(10)-\ln(2)}{\ln(3)}$$
$$= \frac{\ln(10/2)}{\ln(3)} = \frac{\ln 5}{\ln 3} \simeq 1{,}46$$

$$\frac{2\cdot 3^x}{2} = \frac{10}{2}$$
$$3^x = 5$$
$$x\cdot\ln(3) = \ln(5)$$
$$x = \ln(5)/\ln 3 \simeq 1{,}46$$

$$\ln(3^x - 7) \neq \ln(3^x) - 7\ !$$

$$2^x = 3^{x+1}$$
$$\ln(2^x) = \ln(3^{x+1})$$
$$x\cdot\ln(2) = (x+1)\cdot\ln(3)$$
$$x\cdot\ln(2) = x\cdot\ln(3) + \ln(3)$$
$$x\ln(2) - x\ln(3) = \ln(3)$$
$$x\cdot(\ln(2)-\ln(3)) = \ln(3)$$
$$x = \frac{\ln 3}{\ln(2)-\ln(3)} \simeq \frac{1{,}099}{0{,}693-1{,}099} \simeq -2{,}71$$

$$2^x = 3^x - 7$$
$$\cancel{\ln(2^x) = \ln(3^x-7)}$$
$$x\cdot\ln 2 =$$
ikke lett å løse

$$\log_2(x) = 5$$
$$2^{\log_2(x)} = 2^5$$
$$x = 32$$