BI NORWEGIAN BUSINESS SCHOOL

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FORELESNING 11

MET 1180

MATEMATIKK

EIVIND ERIKSEN

Nov 02, 2016

Plan:

1. Tangenter og den deriverte
2. Derivasjonsregler

Pensum:

[E] 4.1-4.3

Merk: Ingen forelesning fredag → fredag 18/11
" kontortid torsdag

Eksamen:

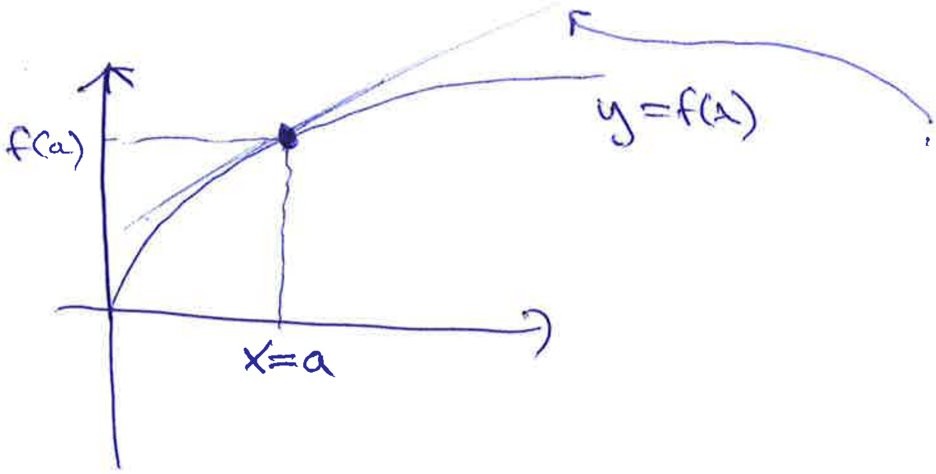
MET 11801 V
MET 11802 : flervalgseksamen 7/12/2016 30%
MET 11803 : mai/juni 70%

↓

15 oppgaver

3p : riktig svar
−1p : galt svar
0p : ikke besvart

1 Tangenter og den deriverte



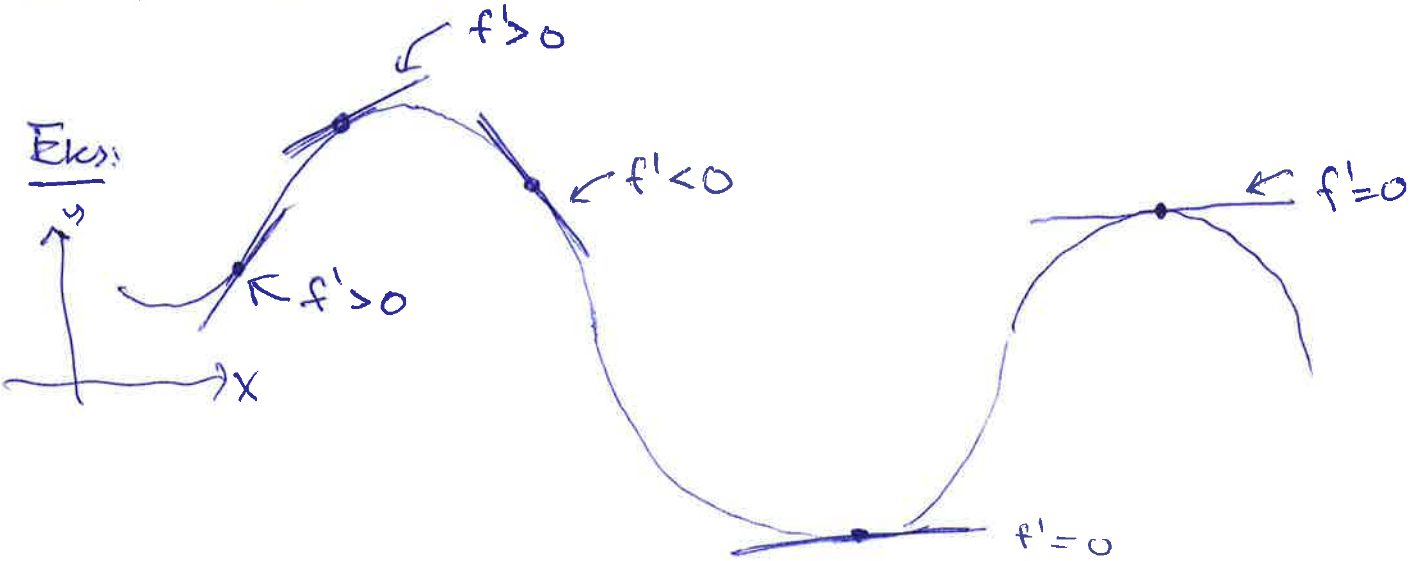
Tangenten til f i punktet x=a:

Den rette linjen gjennom (a, f(a)) som tilnærmer f(a) best mulig i nærheten av x=a.

Den deriverte:

Den deriverte til f i $x=a$ skrives $f'(a)$, og er lik stigningstallet til tangenten til f i $x=a$.

Eks:



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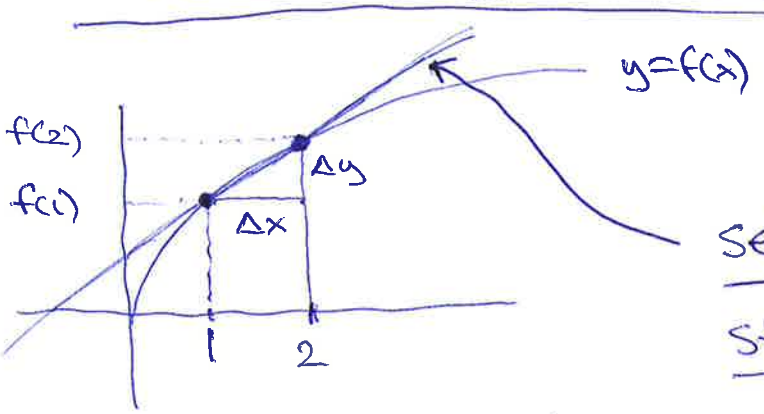
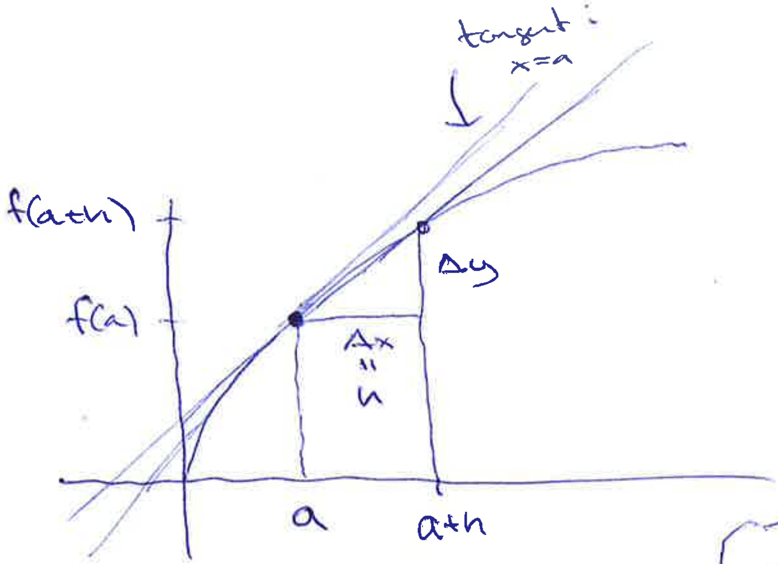
Sekanter og gjennomsnittlig veksthastighet



Sekant gjennom $(1, f(1))$ og $(2, f(2))$

Stigningstall:

$$\frac{\Delta y}{\Delta x} = \frac{f(2)-f(1)}{2-1} = \frac{f(2)-f(1)}{1}$$



$$\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{(a+h)-a}$$

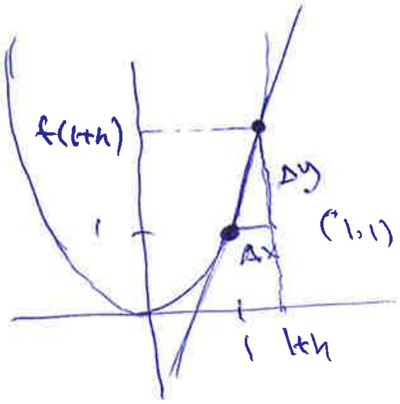
$$= \frac{f(a+h)-f(a)}{h} \approx f'(a)$$

Tilnærmelsen

$$f'(a) \simeq \frac{f(a+h)-f(a)}{h}$$

er bedre jo mindre h er.

$$\boxed{\text{Defn: } f'(a) = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}}$$



Eks: $f(x) = x^2$ i $x = 1$

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} (2+h) = \underline{\underline{2}}$$

den deriverte regnet ut fra definisjonen

$$\frac{\Delta y}{\Delta x} = \frac{f(1+h) - f(1)}{h} = \frac{(1+h)^2 - 1}{h} = \frac{\cancel{1} + 2h + h^2 - \cancel{1}}{h} = \frac{2h + h^2}{h} = \underline{2+h}$$

Stigningstallet til sekanten gjennom $(1,1)$, $(1+h, f(1+h))$

Gjennomsnittlig veksthastighet:

i intervallet x i $[a,b]$ er gjennomsnittlig veksthastighet lik sekantens stigningstall $\frac{f(b)-f(a)}{b-a}$

Momentan veksthastighet i $x=a$: $f'(a)$

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2 Den deriverte funksjonen og derivasjonsregler

Den deriverte i punktet $x=a$: $f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$

et tall

Den deriverte funksjonen: $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

funksjon

Eks: $f(x)=x^2$

$f'(1)=2$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2-x^2}{h} = \frac{\cancel{x^2}+2xh+h^2-\cancel{x^2}}{h} = \underline{2x+h}$$

$$= \lim_{h\to 0} 2x+h = \underline{2x}$$

Konklusjon: $f(x)=x^2 \Rightarrow f'(x)=2x$ den deriverte funksjonen

$f'(1)=2$

$f'(2)=4$

$\vdots$

$f'(0)=0$

$f'(-1)=-2$

$\vdots$

Andre skrivemåter:

$$f'(x) = \frac{df}{dx} = (x^2)' = 2x$$

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Derivasjonsregler:

① $(x^n)' = n \cdot x^{n-1}$ for alle tall n

Eks: $(x^2)' = 2x^1 = \underline{2x}$

$(x^3)' = \underline{3x^2}$

$\left(\frac{1}{x}\right)' = (x^{-1})' = -1 \cdot x^{-2} = \underline{-\frac{1}{x^2}}$

$\left(\frac{1}{x^2}\right)' = (x^{-2})' = -2 \cdot x^{-3} = \underline{-\frac{2}{x^3}}$

$(\sqrt{x})' = (x^{1/2})' = \frac{1}{2} \cdot x^{1/2 - 1} = \frac{1}{2} \cdot x^{-1/2}$

$= \frac{1}{2} \cdot \frac{1}{(x)^{1/2}} = \underline{\underline{\frac{1}{2\sqrt{x}}}}$

Viktige spesial-tilfeller:

$$\boxed{\left(\frac{1}{x}\right)' = -\frac{1}{x^2} \qquad (\sqrt{x})' = \frac{1}{2\sqrt{x}}}$$

② $(u(x) + v(x))' = u'(x) + v'(x) \longleftrightarrow (u+v)' = u' + v'$

$(u(x) - v(x))' = u'(x) - v'(x) \qquad (u-v)' = u' - v'$

Eks: $(x^2 + x + 1)' = (x^2)' + (x)' + (1)' = 2x + 1 \cdot x^0 + 0$

$= \underline{2x + 1}$

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3 $\left(c \cdot u(x)\right)' = c \cdot u'(x)$ når c er en konstant $\Longleftrightarrow$ $(c \cdot u)' = c \cdot u'$ når c er konstant

Eks: $(3x^2)' = 3 \cdot (x^2)' = 3 \cdot 2x = 6x$

$(3 + x^2)' = (3)' + (\cancel{x^2})' = 0 + 2x = 2x$

$\left(3x \cdot (x+1)\right)' = 3 \cdot \left(x(x+1)\right)' = 3 \cdot (x^2 + x)'$

$= 3 \cdot (2x + 1) = 6x + 3$

Viktig spesialtilfelle: $\boxed{(ax+b)' = a}$ a, b konstanter

4 Produktregel: $\left(u(x) \cdot v(x)\right)' = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

$(u \cdot v)' = u'v + uv'$

Eks: $(xe^x)' = (x)' \cdot e^x + x \cdot (e^x)'$

$= 1 \cdot e^x + x \cdot (e^x)' = e^x + x \cdot e^x$

$(x^1 \cdot x^{1/2})' = (x \cdot \sqrt{x})' = 1 \cdot \sqrt{x} + x \cdot \frac{1}{2\sqrt{x}}$

$= \sqrt{x} + \frac{x}{2\sqrt{x}} = \sqrt{x} + \frac{\sqrt{x}}{2}$

$\frac{x}{\sqrt{x}} = x^{1-1/2} = x^{1/2} = \sqrt{x}$

$= \frac{3}{2}\sqrt{x}$

$(x^1 \cdot x^{1/2})' = (x^{3/2})' = \frac{3}{2} \cdot x^{1/2} = \frac{3}{2}\sqrt{x}$

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Derivasjon av e^x og $\ln(x)$:

Eks: $f(x) = a^x$

$$f'(x) = \lim_{h\to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h\to 0} \frac{a^x \cdot a^h - a^x}{h}$$

$$= \lim_{h\to 0} \frac{a^x \cdot (a^h - 1)}{h} = a^x \boxed{\lim_{h\to 0} \frac{a^h - 1}{h}}$$

et tall som avhenger av a.

$(2^x)' \simeq 2^x \cdot 0{,}693$ ← $\underline{a=2}$: $\lim_{h\to 0} \frac{2^h - 1}{h} \simeq 0{,}693$

$(3^x)' \simeq 3^x \cdot 1{,}099$ ← $\underline{a=3}$: $\lim_{h\to 0} \frac{3^h - 1}{h} \simeq 1{,}099$

$a = e = 2{,}71828\ldots$ Euler's tall

$\Downarrow$

$(a^x)' = a^x \cdot 1$

⑤ $$\boxed{\begin{aligned} (e^x)' &= e^x \\ (a^x)' &= a^x \cdot \ln(a) \end{aligned}}$$

Det viser seg at $\lim_{h\to 0} \frac{a^h - 1}{h} = \ln(a)$ for alle $a > 0$.

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6 $(\ln(x))' = 1/x$

$(\log_a(x))' = \frac{1}{x} \cdot \frac{1}{\ln(a)} \quad (a>0)$

Eks: $(x^2 \cdot \ln x)' = (x^2)' \cdot \ln x + x^2 \cdot (\ln x)'$

$= 2x \cdot \ln x + x^2 \cdot \frac{1}{x}$

$= \underline{2x \ln x + x}$

Viktige spesial-tilfeller: $\boxed{(e^x)' = e^x \qquad (\ln(x))' = 1/x}$

7 Brøkregel: $\left(\frac{u(x)}{v(x)}\right)' = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v(x)^2}$

(kvotientregelen) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

Eks: $\left(\frac{x+1}{x-2}\right)' = \frac{(x+1)' \cdot (x-2) - (x+1) \cdot (x-2)'}{(x-2)^2}$

$= \frac{1 \cdot (x-2) - (x+1) \cdot 1}{(x-2)^2} = \frac{\cancel{x} - 2 - \cancel{x} - 1}{(x-2)^2}$

$= \underline{\underline{\frac{-3}{(x-2)^2}}}$

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Eks: $\left(\dfrac{x^2-2x+3}{x^3}\right)' = \left(x^{-1} - 2x^{-2} + 3x^{-3}\right)'$

$u = x^2-2x+3$
$v = x^3$

$= -x^{-2} - 2\cdot(-2)x^{-3} + 3(-3)x^{-4}$

$= -\dfrac{1}{x^2} + \dfrac{4}{x^3} - \dfrac{9}{x^4}$

$$= \frac{\overset{u'}{(2x-2)}\cdot \overset{v}{x^3} - (x^2-2x+3)\cdot 3x^2}{x^6} = \frac{x^2\left[x(2x-2)-3(x^2-2x+3)\right]}{x^6}$$

$$= \frac{2x^2-2x-3x^2+6x-9}{x^4} = \frac{-x^2+4x-9}{x^4} = -\frac{1}{x^2}+\frac{4}{x^3}-\frac{9}{x^4}$$

Derivasjon av sammensatte funksjoner:

Eks: $f(x) = \dfrac{1}{(1-x)^2} = (1-x)^{-2} = u^{-2}$, $u = 1-x$ kjerne

$\boxed{f'(x) = f'(u)\cdot u'(x)}$ kjerneregelen

$f'(x) = (u^{-2})'_u \cdot (1-x)'_x = -2\cdot u^{-3}\cdot(-1)$

$= 2\cdot u^{-3} = \dfrac{2}{u^3} = \dfrac{2}{(1-x)^3}$

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(8) Kjerneregelen:

Når $f(x) = f(u)$ med kjerne $u = u(x)$,

da er

$$\boxed{\begin{aligned} f'(x) &= f'(u) \cdot u'(x) \\ \frac{df}{dx} &= \frac{df}{du} \cdot \frac{du}{dx} \end{aligned}}$$

Eks: $f(x) = \sqrt{x^2+1} = \sqrt{u}$, $\quad u = x^2+1$

$$f'(x) = \frac{df}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{2\sqrt{u}} \cdot 2x = \frac{\cancel{2}x}{\cancel{2}\sqrt{u}} = \frac{x}{\sqrt{x^2+1}}$$

$$f(x) = xe^{-x}$$

$$\boxed{\begin{aligned} e^{-x} &= e^{u}, \quad u = -x \\ (e^{-x})' &= e^{u} \cdot (-1) \end{aligned}}$$

$$f'(x) = 1 \cdot e^{-x} + x \cdot (e^{-x})'$$

$$= e^{-x} + x\left(e^{-x} \cdot (-1)\right)$$

$$= e^{-x} - xe^{-x}$$

Derivasjonsregler:

① $(x^n)' = nx^{n-1}$

② $(u \pm v)' = u' \pm v'$

$(c \cdot u)' = c \cdot u'$

③ $(u \cdot v)' = u' \cdot v + u \cdot v'$

④ $(u/v)' = \frac{u'v - uv'}{v^2}$

⑤ $(h(u(x)))' = h'(u(x)) \cdot u'(x)$

$= \frac{dh}{du} \cdot \frac{du}{dx}$

⑥ $(e^x)' = e^x$

$(a^x)' = a^x \cdot \ln(a) \qquad (a > 0)$

⑦ $(\ln(x))' = \frac{1}{x}$

$(\log_a(x))' = \frac{1}{x} \cdot \frac{1}{\ln(a)} \qquad (a > 0)$

$u = u(x)$, $v = v(x)$

- Vilk. uttrykk i x

c, n

- konstanter