BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

FORELESNING 13

MET 1180

MATEMATIKK

EIVIND ERIKSEN   NOV 16 2016

Plan:

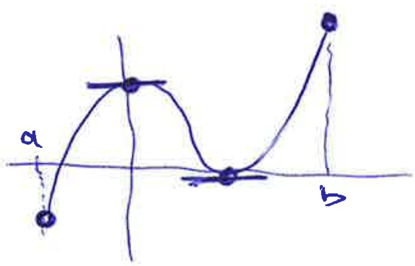
1. Globale maks/min
2. Andre deriverte og krumning
3. Konvekse/konkave funksjoner

Pensum:

[E] 4.6 – 4.7

Fredag: Foreles. 11–14

Repetisjon: Lokale maks/min for $f(x)$

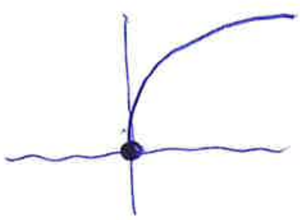


x lokalt maks/min for f $\Rightarrow$ x er enten:

i) Stasjonært pkt $f'(x)=0$

ii) randpkt $D_f=[a,b]$, $x=a$, $x=b$

iii) kritisk pkt der $f'(x)$ ikke eksisterer



Punktene i), ii), iii) kalles kandidatpunkter.

BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

Eks: $f(x) = x^2 e^x$, $D_f =$ [-2, 4]

Kandidatpunkter: $x = -2$, $x = 0$, $x = 4$

i) Stasjonære pkt:

$$f'(x) = 0$$
$$(x^2 e^x)' = 0$$
$$2x \cdot e^x + x^2 e^x = 0$$
$$(2x + x^2)e^x = 0$$
$$2x + x^2 = 0$$
$$x \cdot (2 + x) = 0$$
$$x = 0 \quad , \quad x = -2$$

ii) Randpkt: $x = -2$, $x = 4$

iii) Kritisk pkt der $f'(x)$ ikke eks.

$f'(x) = (2x + x^2)e^x$

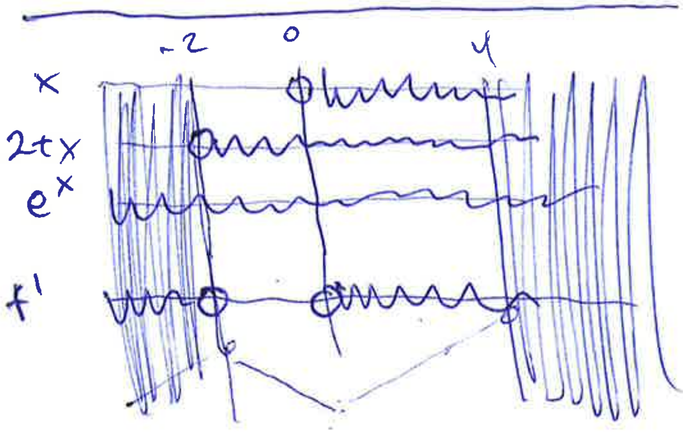
Ingen slike pkt.

$x = 0$: $f(0) = 0$

$x = -2$: $f(-2) = 4e^{-2} = \frac{4}{e^2}$

$x = 4$: $f(4) = 16e^4$

Fortegnsdiagram for $f'(x)$:



$$f'(x) = (2x + x^2)e^x = x \cdot (2 + x) e^x$$

$x = 0$: lokalt min

$x = -2, 4$: lokale maks

(1) Hvordan finne maks/min = globale maks/min

globalt maks/min $\Rightarrow$ lokalt maks/min $\Rightarrow$ kandidatpkt
i), ii), iii)

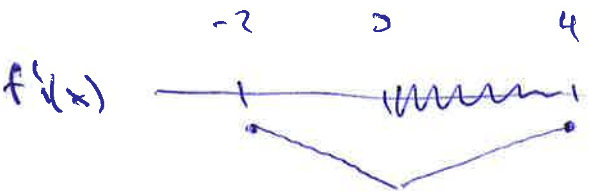
Eks: $f(x) = x^2 e^x$, $D_f = [-2, 4]$

Kandidatpunkt: $x = -2$, $x = 0$, $x = 4$

$f(-2) = \frac{4}{e^2}$ $f(0) = 0$ $f(4) = 16e^4$

Globalt maks: Beste kandidat $x = 4$ $f(4) = 16e^4$
= maks

Fortegnsdiagram for $f'(x)$:

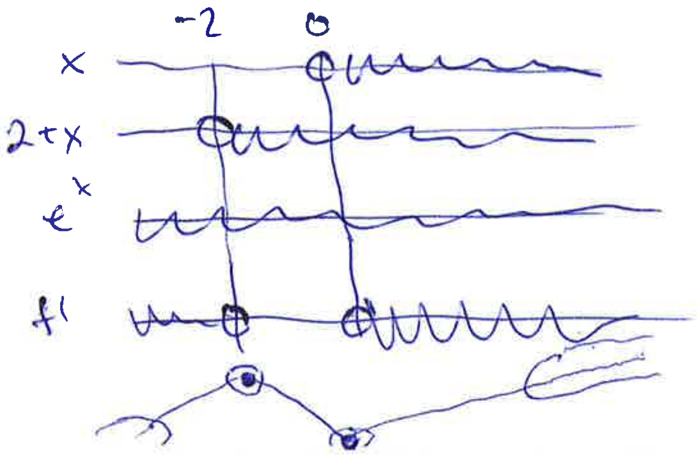


$x = 4$ er globalt maks

Globalt min $x = 0$
= min.

Eks: $f(x) = x^2 e^x$, $D_f = (\leftarrow, \rightarrow) = \mathbb{R}$

$f'(x) = (2x + x^2)e^x = (2+x)x e^x$



Kandidatpkt:

$x = -2$, $x = 0$

Globalt maks

$f(-2) = \frac{4}{e^2}$
$\lim_{x \to \infty} f(x) = \infty$
ingen globale maks

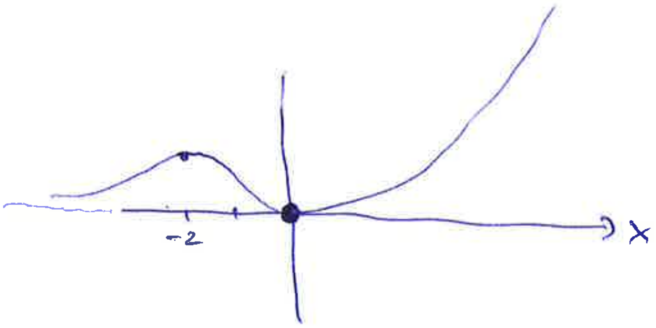
BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

Globalt min: $x = 0$ $\quad f(0) = 0$

$x \to -\infty$ $\quad \lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} x^2 e^x$

$= \lim\limits_{t\to\infty} (-t)^2 e^{-t} = \lim\limits_{t\to\infty} t^2/e^t = 0$

$\boxed{t = -x}$



Konklusjon: $x=0$ er globalt min

Eks: $\quad f(x) = (x^2+x-5)e^{-x}$, $D_f = \mathbb{R}$

Finn globale maks/min, hvis de fins.

$f'(x) = (2x+1)e^{-x} + (x^2+x-5)\cdot e^{-x}\cdot(-1)$

$= e^{-x}\left[2x+1-x^2-x+5\right]$

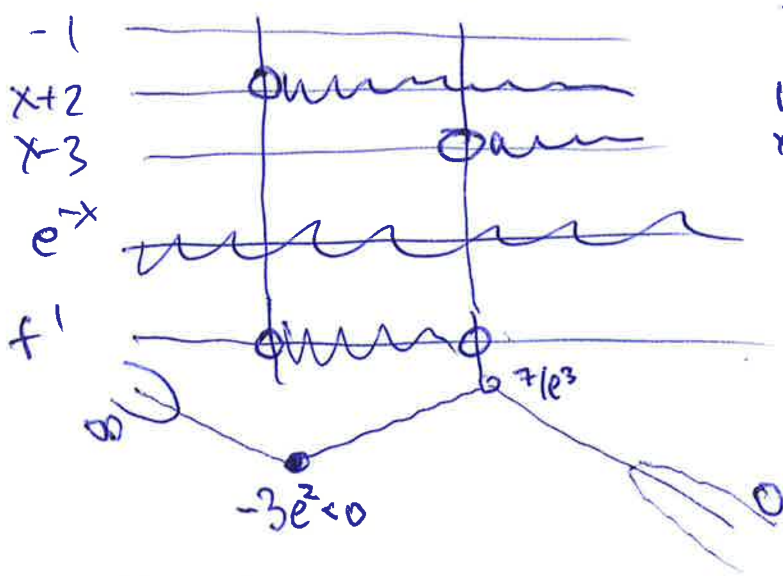
$= e^{-x}\left[-x^2+x+6\right]$

$= -(x+2)(x-3)e^{-x}$

$-x^2+x+6=0$

$x = \dfrac{-1\pm\sqrt{1-4\cdot(-1)\cdot 6}}{-2}$

$= \dfrac{-1\pm 5}{-2} = -2, 3$



$x=-2$: $f(-2) = -3e^2$

$\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} \dfrac{x^2+x-5}{e^x}$

$= 0$

$\Downarrow$

$x=-2$ globalt min

$x=3$: $f(3) = 7e^{-3}$
$= \dfrac{7}{e^3}$

$\lim\limits_{x\to-\infty} f(x) =$
$\lim\limits_{x\to-\infty} (x^2+x-5)e^{-x}$
$= \infty$

ingen globale maks

BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

Ekstremverdisetningen:

Hvis f er en kontinuerlig funksjon definert på et lukket, begrenset intervall $[a,b]$, så har f et globalt maks og et globalt min.

(Tilsvarende gjelder også endelige unioner av lukkede begrensede intervaller)

Ok for $D_f = [3, 8]$

$D_f = [0,1] \cup [3,7] \cup [10,12]$

~~Ikke ok for:~~ $\cancel{D_f = [0, \rightarrow)}$

$\cancel{D_f = [0,1)}$

BI NORWEGIAN BUSINESS SCHOOL

Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo

Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00

www.bi.edu
study@bi.no
Org. no. 971 22 8865

(2) Høgere ordens deriverte

Andrederiverte: $f''(x)$

Eks: $f(x) = x^2$

$f'(x) = 2x$

$f''(x) = (2x)' = \underline{2}$

$f(x) = xe^{-x}$

$f'(x) = 1 \cdot e^{-x} + x \cdot e^{-x} \cdot (-1)$

$= \underline{(1-x)e^{-x}}$

$f''(x) = \left((1-x)e^{-x}\right)'$

$= -1 \cdot e^{-x} + (1-x)e^{-x} \cdot (-1)$

$= e^{-x}(-1-1+x)$

$= \underline{\underline{(x-2)e^{-x}}}$

Høyere ordens deriverte:

$f'''(x)$ = tredjederivert

$f^{(4)}(x)$ = fjerdederivert

$\vdots$

Eks: $f(x) = x^3 - x + 2$

$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

$f'''(x) = 6$

$f^{(4)}(x) = 0$

$f^{(5)}(x) = 0$

$\vdots$

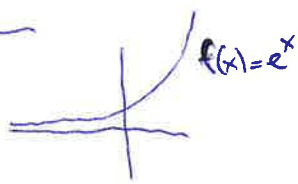
(3) Andrederiverte som krumning.

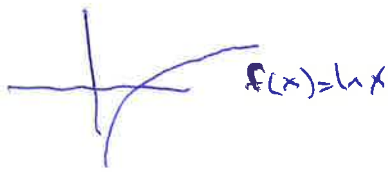
Eks:

i) $f(x) = x$ $\quad f' = 1 > 0$ voksende

ii) $f(x) = e^x$ $\quad f' = e^x > 0$ voksende

iii) $f(x) = \ln x,\ x > 0$ $\quad f' = 1/x > 0$ voksende





BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

Tolkning av $f''(x)$:

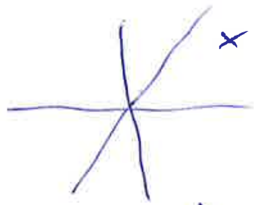
$$f''(x) = (f'(x))' = \text{veksten i } f'(x)$$

$f'' > 0$: f' vokser

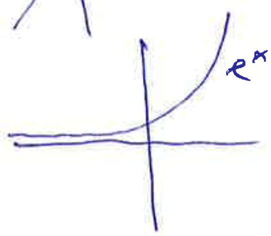
konveks

$f'' < 0$: f' avtar

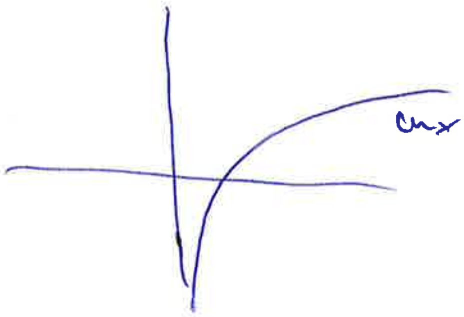
i) $f(x) = x \qquad f'(x) = 1 \qquad f''(x) = 0$



ii) $f(x) = e^x \qquad f'(x) = e^x \qquad f''(x) = e^x > 0$



iii) $f(x) = \ln x$, $x > 0 \qquad f'(x) = 1/x \qquad f''(x) = -\frac{1}{x^2} < 0$



__Defn:__ f kalles __konveks__ i $[a,b]$ dersom $f''(x) \geq 0$ for alle x i (a,b)

f kalles __konkav__ i $[a,b]$ dersom $f''(x) \leq 0$ for alle x i (a,b)

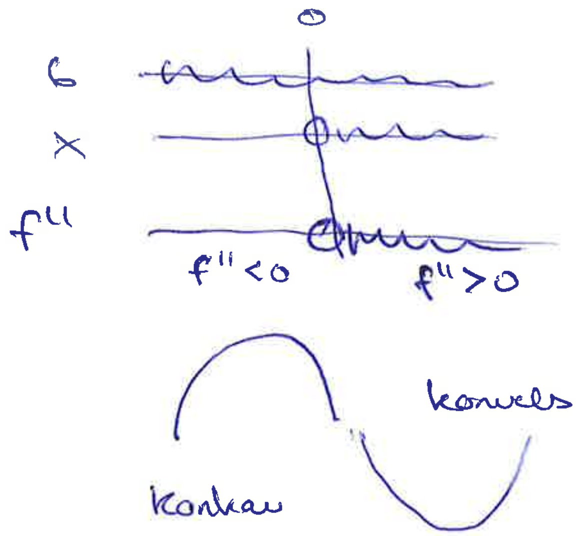
BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

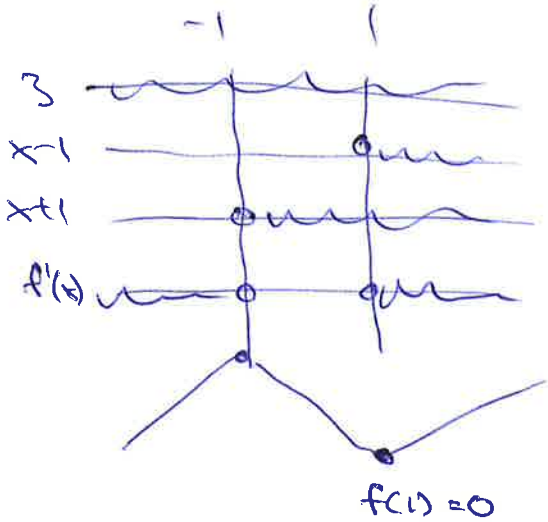
Eks:

$$f(x) = x^3 - 3x + 2$$

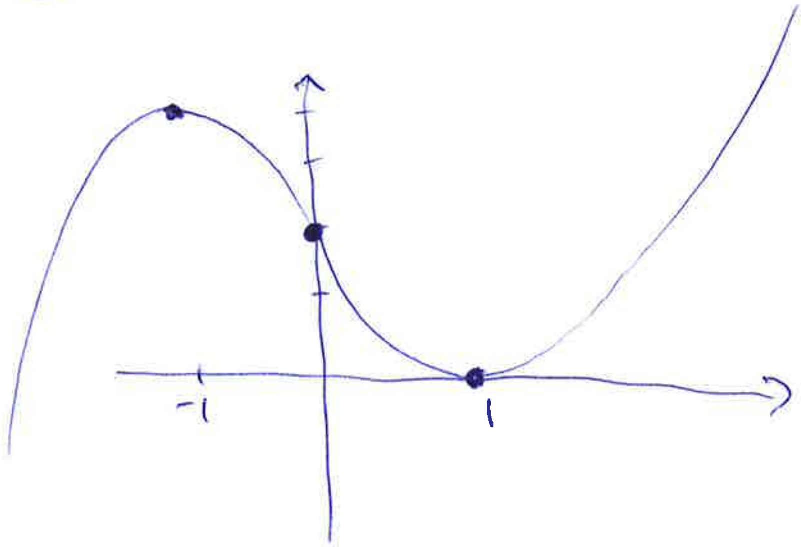
$$f'(x) = 3x^2 - 3 = 3(x^2 - 1) = 3(x-1)(x+1)$$

$$f''(x) = 6x$$





x=0 kalles et vendepunkt



$\boxed{\text{Defn: En } \underline{\text{vendepkt}}\ x = x^* \text{ for } f \text{ er et pkt i } D_f \text{ der } f''(x) \text{ skifter fortegn.}}$

Finner vendepkt ved å sette opp fortegnslinja for $f''(x)$.

<u>Vendetangent</u> = tangent i vendepkt.

<u>Eks:</u>

$$f(x) = x^2 e^{-x}$$

$$f'(x) = 2x \cdot e^{-x} + x^2 \cdot e^{-x} \cdot (-1) = (2x - x^2)\, e^{-x}$$

$$f''(x) = (2-2x)e^{-x} + (2x-x^2) \cdot e^{-x} \cdot (-1)$$

$$= (2 - 2x - 2x + x^2) \cdot e^{-x}$$

$$= \underline{(x^2 - 4x + 2)\, e^{-x}} = (x - x_1)(x - x_2)\, e^{-x}$$

$$x^2 - 4x + 2 = 0$$

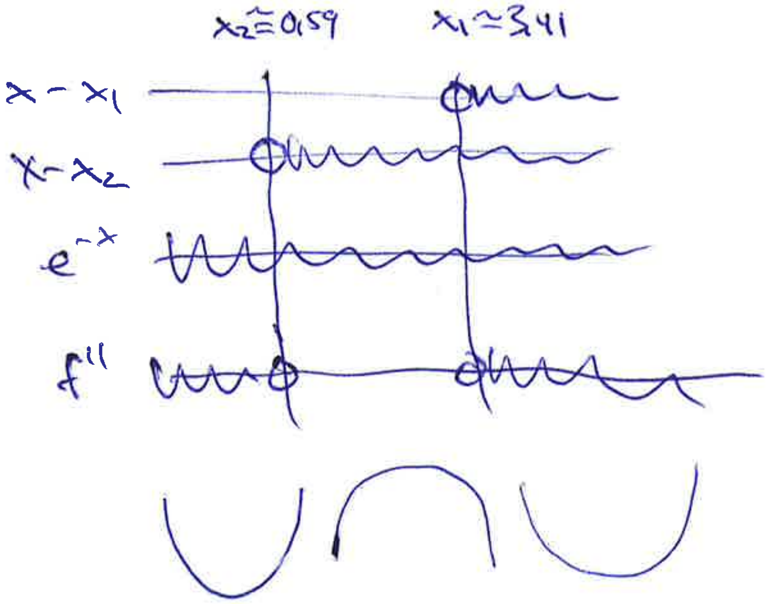
$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 2}}{2}$$

$$= 2 \pm \frac{\sqrt{8}}{2} = 2 \pm \frac{\cancel{\sqrt{4}} \cdot \sqrt{2}}{\cancel{2}}$$

$$= 2 \pm \sqrt{2}$$

$$x_1 = 2 + \sqrt{2} \simeq \underline{3,41}$$

$$x_2 = 2 - \sqrt{2} \simeq \underline{0,59}$$



<u>Vendepkt:</u> $x = x_1 \simeq 3,41$

$x = x_2 \simeq 0,59$

BI NORWEGIAN BUSINESS SCHOOL
Nydalsveien 37, 0484 Oslo
Mailing address NO-0442 Oslo
Telephone [+47] 46 41 00 00
Telefax [+47] 21 04 80 00
www.bi.edu
study@bi.no
Org. no. 971 22 8865

Defn: Hvis f er kontinuerlig på intervallet $[a,b]$, så kalles f:

konveks hvis $f''(x) \geq 0$ for alle x i (a,b)

konkav " $f''(x) \leq 0$ —"—

Defn: f kalles **konveks** hvis f er **konveks** i hele D_f, og f kalles **konkav** hvis f er konkav i hele D_f.

Eks:

$$f(x) = x^2e^x - x + 1$$

$$f'(x) = 2xe^x + x^2 \cdot e^x - 1 = (2x+x^2)e^x - 1$$

$$f''(x) = (2+2x)e^x + (2x+x^2)e^x$$

$$= (2+4x+x^2)\,e^x$$

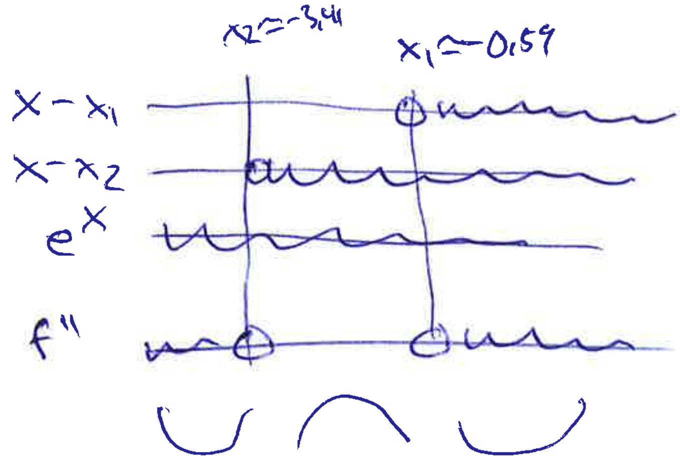
$$x^2+4x+2=0$$

$$x = \frac{-4 \pm \sqrt{16-8}}{2}$$

$$= -2 \pm \sqrt{2}$$

$$x_1 = -2+\sqrt{2} \cong -0{,}59$$

$$x_2 = -2-\sqrt{2} \simeq -3{,}41$$

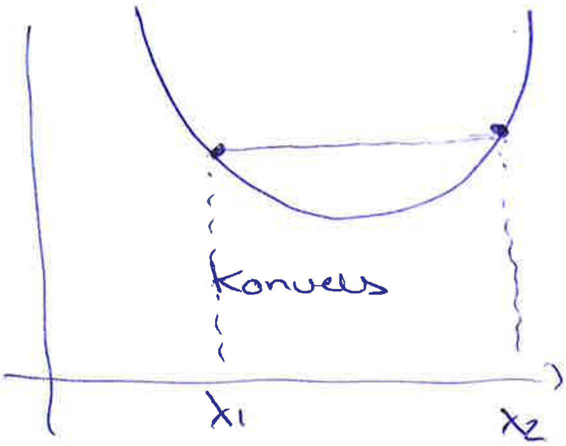


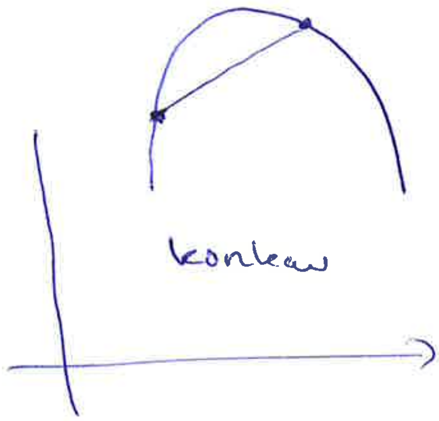
to **vendepkt**

Krumning:

$f'' > 0$

$f'' < 0$





Konveks: det rette linjestykket gjennom to pkt på grafen ligger over grafen

Konkav: det rette linjestykket gjennom to pkt på grafen ligger under grafen.

Eks: $f(x) = x^3 - 4x^2 + 5x - 2$

$$f'(x) = 3x^2 - 8x + 5 =$$

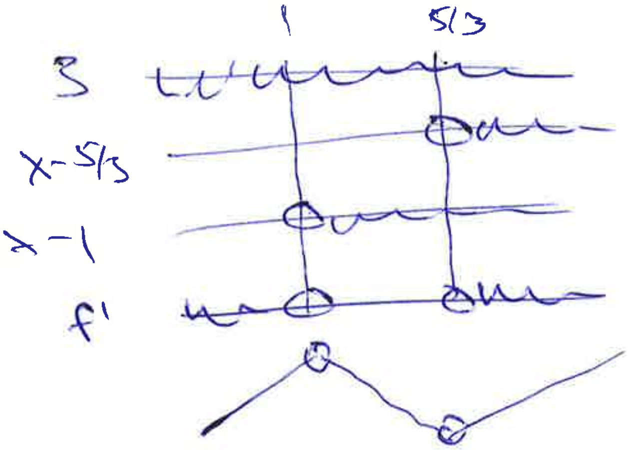
Fortegnsskjema for $f'(x)$:

$$3x^2 - 8x + 5 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 4\cdot 3\cdot 5}}{6}$$

$$= \frac{8 \pm 2}{6} = \frac{10}{6}, 1$$

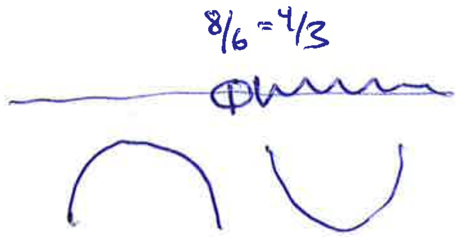
$$f'(x) = 3\left(x - \frac{5}{3}\right)(x-1)$$



f voksende i $(\leftarrow, 1]$ og i $[5/3, \rightarrow)$
" avtagende i $[1, 5/3]$

Lokale maks: $x = 1$ $f(1) = 0$
Lokale min: $x = 5/3$ $f(5/3) = \ldots$

$f''(x) = 6x - 8$ f''



$$f''(x) = 0$$

$$6x - 8 = 0$$

$$x = 8/6 = 4/3$$

f konveks i $[4/3, \rightarrow)$
" konkav i $(\leftarrow, 4/3]$

Vendepkt: $x = 4/3$