FORELESNING

MET1180

MATEMATIKK

EIVIND ERIKSEN

DES 11, 2017

Plan:

1. Gjennomgang av eksamen MET11802 fra 12/2016
2. Repetisjon av viktige temaer

Pensum: Kap 1–4 i læreboken
Oppgaver inkl. innleveringsoppgaver

1. Ved årsslutt:

$$200.000 \cdot \underbrace{\left(1 + \frac{0.02}{4}\right)}_{1,005} = 201.000 \leftarrow 31.12.2015$$

Etter x år:

$$201.000 \cdot 1,02^x = 250.000$$

$$1,02^x = \frac{250}{201}$$

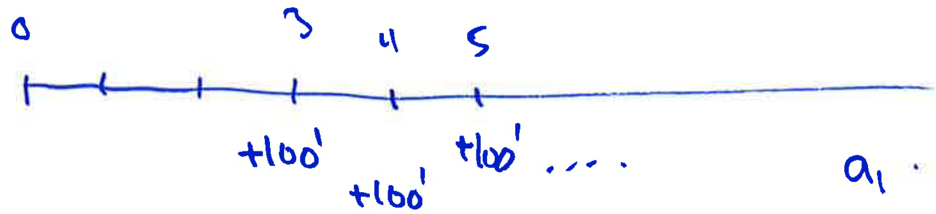
$$\ln(1,02^x) = \ln\left(\frac{250}{201}\right)$$

$$x \cdot \ln 1,02 = \ln(250/201)$$

$$x = \frac{\ln(250/201)}{\ln 1,02} \simeq 11,02 \rightarrow \underline{12 \text{ år}}$$

Dus: $\underline{\underline{31/12/2027}}$ c

2.



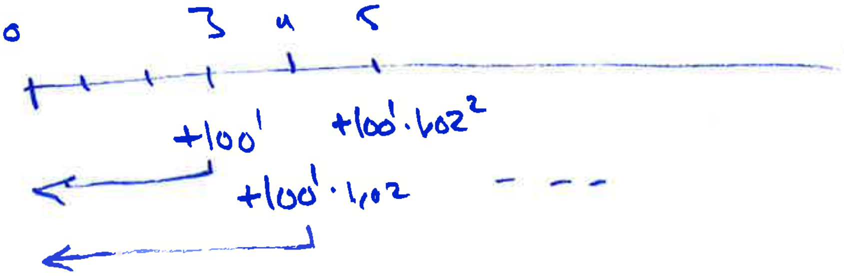
$$a_1 \cdot \frac{1}{1-k}$$

Nåverdi:

$$\frac{100.000}{1,10^3} + \frac{100.000}{1,10^4} + \dots = \frac{100.000}{1,10^3} \cdot \frac{1}{1 - \frac{1}{1,10}}$$

$$= \frac{100.000}{1,10^2 \cdot 0,10} \simeq 826.446 \text{ kr}$$ c

(3)

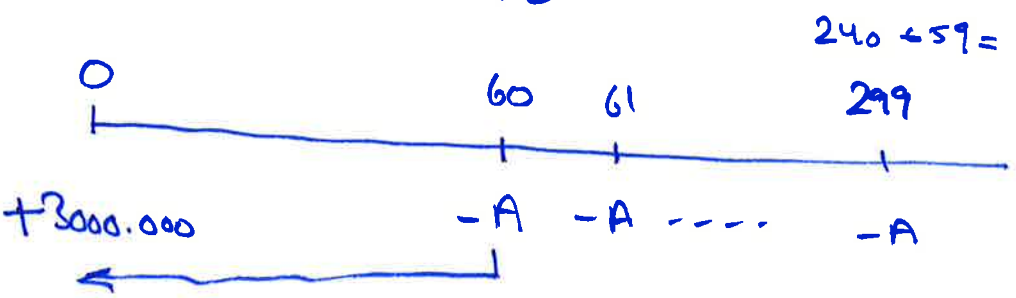


$$\frac{100.000}{1,10^3} + \frac{100.000\cdot 1,02}{1,10^4} + \frac{100.000\cdot 1,02^2}{1,10^5} + \dots$$

$$= \frac{100.000}{1,10^3}\cdot\frac{1}{1-\frac{1,02}{1,10}} = \frac{100.000}{1,10^2\,(1,10-1,02)} \simeq 1.033.058\text{ kr}$$

D

(4)



$$r = \frac{2,40\%}{12} = 0,2\% = 0,002$$

$$3000.000 - \frac{A}{1,002^{60}} - \frac{A}{1,002^{61}} - \dots - \frac{A}{1,002^{299}} = 0$$

$$3000.000 = \frac{A}{1,002^{60}} + \dots + \frac{A}{1,002^{299}} \qquad k = \frac{1}{1,002}$$

$$= \frac{A}{1,002^{60}}\cdot\frac{1-\left(\frac{1}{1,002}\right)^{240}}{1-\frac{1}{1,002}} = \frac{A\,(1,002^{240}-1)}{1,002^{59}\cdot 0,002\cdot 1,002^{240}}$$

$$A = \frac{3.000.000\cdot 0,002\cdot 1,002^{299}}{(1,002^{240}-1)} \simeq 17.722,02\text{ kr}$$

Samlede renter: $240\cdot A - 3000.000 = \underline{1.253.284\text{ kr}}$

B

5.

$$\frac{6-x}{x^2+4} \geq 1 \xrightarrow{\cdot(x^2+4)} 6-x \geq x^2+4$$

$$-x^2-x+2 \geq 0$$

$\downarrow$ fortegnsskjema

$\vdots$

$$\frac{6-x}{x^2+4} - 1 \geq 0$$

$$\frac{6-x-(x^2+4)}{x^2+4} \geq 0$$

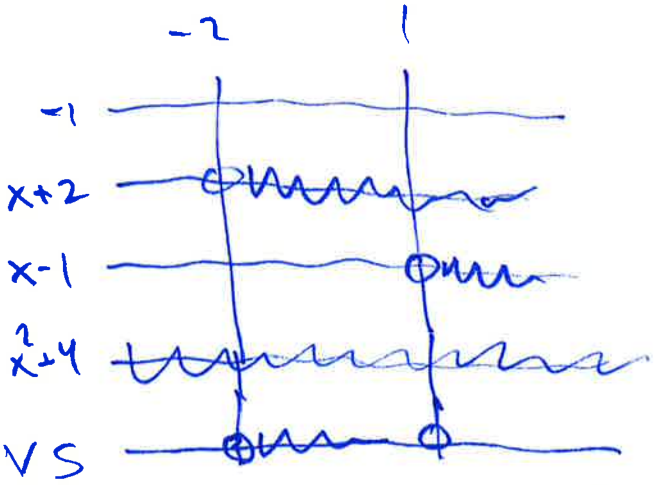
$$\frac{-x^2-x+2}{x^2+4} \geq 0$$

$$-x^2-x+2=0$$

$$x=\frac{1\pm\sqrt{1+8}}{-2}=\frac{1\pm 3}{-2}$$

$$= -2,\ 1$$

$$\frac{-(x+2)\cdot(x-1)}{x^2+4} \geq 0$$



$$L=[-2,1]$$

A

6. $\frac{1}{x} = \frac{x}{x+1} \quad | \cdot x(x+1) \quad , x \neq 0, -1$

$$x+1 = x^2$$

$$0 = x^2 - x - 1$$

$$x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$x_1 = \frac{1+\sqrt{5}}{2} > 0 \qquad x_2 = \frac{1-\sqrt{5}}{2} < 0$$

B

7. $\ln x = \sqrt{2\ln x + 4} \qquad u = \ln x$

$$u = \sqrt{2u+4} \quad | \ (\cdot)^2$$

$u > 0$: løsn.
$u < 0$: falsk løsn.

$$u^2 = 2u + 4$$

$$u^2 - 2u - 4 = 0$$

$$\ln x = u = \frac{2 \pm \sqrt{4+16}}{2} = 1 \pm \frac{\sqrt{20}}{2}$$

$\ln x = 1 + \frac{\sqrt{20}}{2}$ løsn.

$\cancel{\ln x = 1 - \frac{\sqrt{20}}{2}}$ ~~falsk løsn.~~

$$x = e^{1+\sqrt{20}/2}$$

B

$$1 + \frac{\sqrt{20}}{2} = \sqrt{2 \cdot \left(1 + \frac{\sqrt{20}}{2}\right) + 4}$$

$$1 - \frac{\sqrt{20}}{2} = \sqrt{2 \cdot \left(1 - \frac{\sqrt{20}}{2}\right) + 4}$$

8. $f(x) = \frac{x^2-3x-1}{x+3} = (x-6) + \frac{17}{x+3}$

BI

vertikal: $x+3=0$ $\quad a=-3$

$x=-3$

skrå: $(x^2-3x-1):(x+3) = x-6$

$-(x^2+3x)$

$-6x-1$

$-(-6x-18)$

17

$\Downarrow$

$y = x-6$ skrå $\quad b=-6$

$a-b = -3-(-6) = 3$ $\quad$ B

9. $f'(0) =$ stign. tall til tangenten i $x=0$

$f(x) = x^2 \cdot \ln(1-x) = u \cdot v$

$f'(x) = u' \cdot v + u \cdot v' = 2x \cdot \ln(1-x) + x^2 \cdot \frac{1}{1-x} \cdot (-1)$

$f'(0) = \underline{\underline{0}}$ $\quad$ A

10.

$$f(x)=\frac{x^2-3x}{x+1}=\frac{u}{v}$$

$$f'(x)=\frac{u'\cdot v-u\cdot v'}{v^2}=\frac{(2x-3)\cdot(x+1)-(x^2-3x)\cdot 1}{(x+1)^2}$$

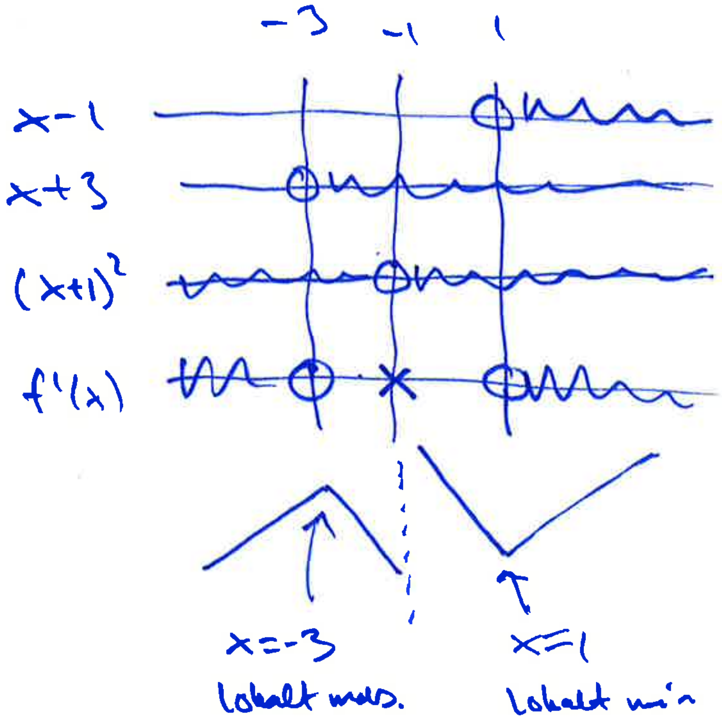
$$=\frac{(2x^2-x-3)-(x^2-3x)}{(x+1)^2}=\frac{x^2+2x-3}{(x+1)^2}$$

$$x^2+2x-3=0$$

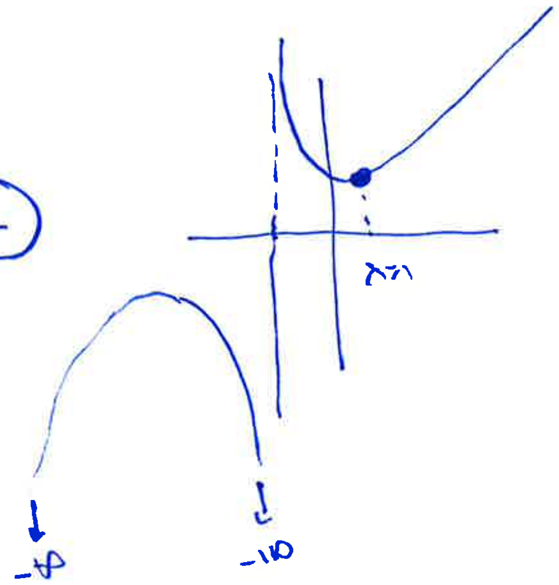
$$x=\frac{-2\pm\sqrt{4+12}}{2}=\frac{-2\pm4}{2}=1,-3$$

Stasjonære pkt.

$$=\frac{(x-1)(x+3)}{(x+1)^2}$$



c)



11.

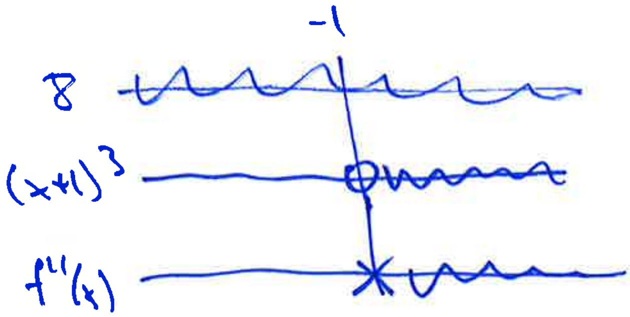
$$f(x) = \frac{x^2-3x}{x+1}$$

$$f'(x) = \frac{x^2+2x-3}{(x+1)^2} = \frac{(x-1)(x+3)}{(x+1)^2} \quad \frac{u}{v}$$

$$f''(x) = \frac{u'v - uv'}{v^2} = \frac{(2x+2)\cdot(x+1)^2 - (x^2+2x-3)\cdot 2\cdot(x+1)\cdot 1}{(x+1)^4}$$

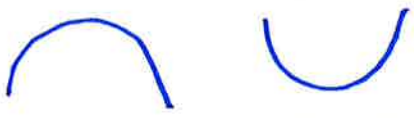
$$= \frac{\cancel{(x+1)}\cdot\left[(2x+2)\cdot(x+1) - (x^2+2x-3)\cdot 2\right]}{(x+1)^{\cancel{4}\,3}}$$

$$= \frac{(\cancel{2x^2}+\cancel{4x}+2)-(\cancel{2x^2}+\cancel{4x}-6)}{(x+1)^3} = \frac{8}{(x+1)^3}$$



$$f(x) = \frac{x^2-3x}{x+1}, \quad \underline{x \neq -1}$$

<u>ingen vendepkt</u> D



konkav konveks

12.

$$D(p) = 110 - 5p$$
$$D'(p) = -5$$

$$El_p\, D(p) = D'(p)\cdot\frac{p}{D(p)}$$

$$\varepsilon = -5 \cdot \frac{p}{110-5p} = -1$$

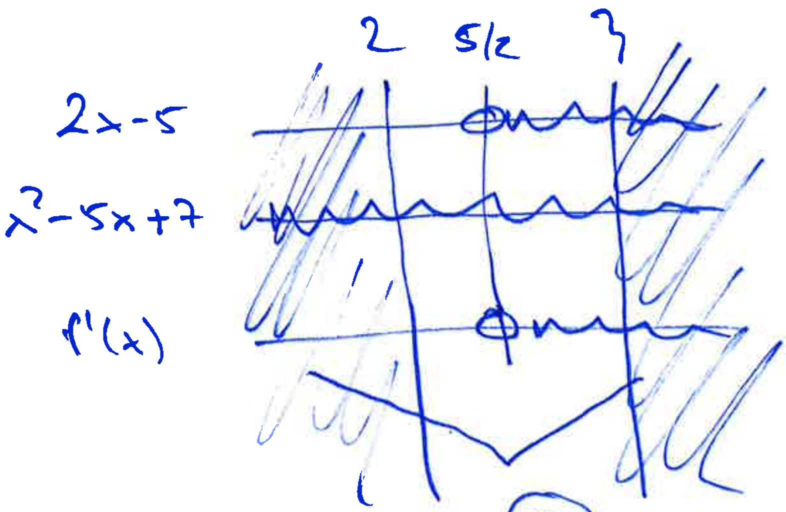
$$\frac{p}{110-5p} = \frac{1}{5} \quad \Big|\,(110-5p)5$$

$$5p = 110 - 5p$$

$$10p = 110 \qquad p = \underline{\underline{11}} \quad B$$

13. $f(x) = \ln(x^2-5x+7) \qquad x \in [2,3]$

$$f'(x) = \frac{1}{x^2-5x+7} \cdot (2x-5) = \frac{2x-5}{x^2-5x+7}$$



$x^2-5x+7=0$

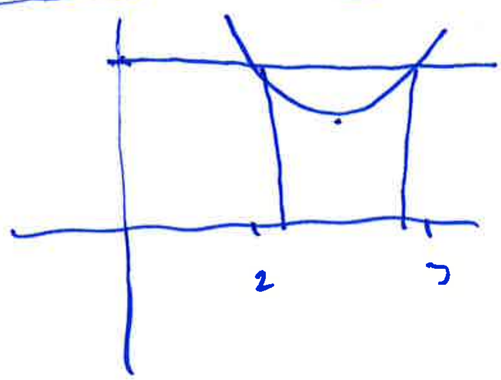
$x = \frac{5 \pm \sqrt{25-28}}{2}$

ingen løsn.

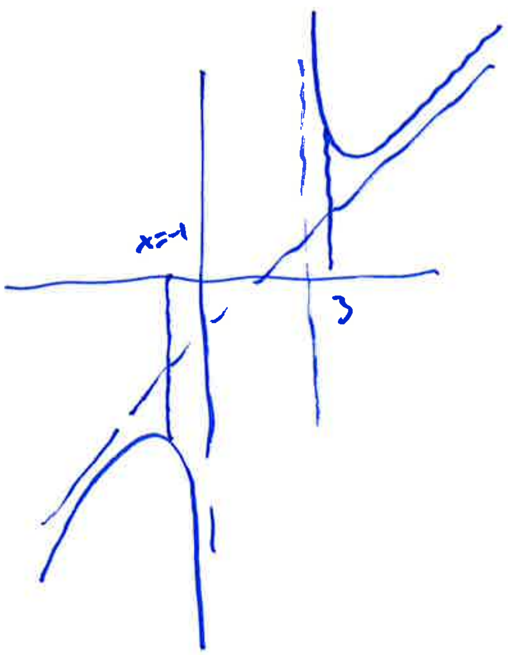
A

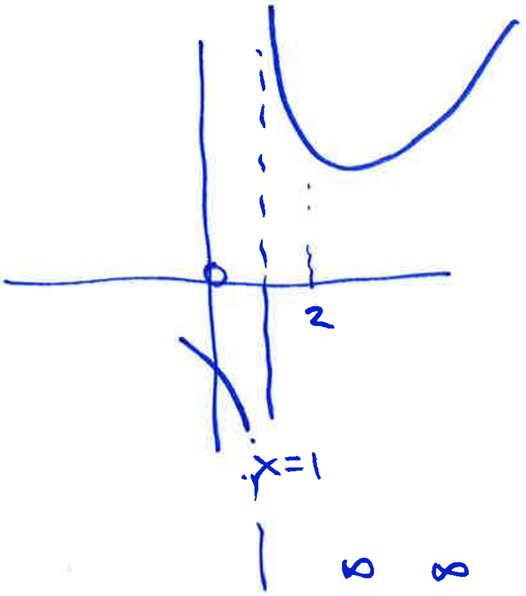
Huis f er kontinuerlig på et intervall:

f har en omvendt funksjon $\Longleftrightarrow$ f er voksende i D_f eller f er avtagende i D_f



$D_f =$





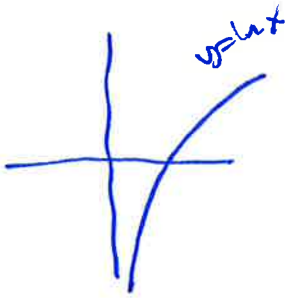
$D_f = [0,1) \cup (1,2]$

14. $\displaystyle \lim_{x\to\infty} \frac{1 - \overset{\infty}{\overset{\uparrow}{x}}\,\overset{\infty}{\overset{\uparrow}{\ln x}}}{\underset{\infty}{e^x}} = \ell \quad \text{"}\frac{-\infty}{\infty}\text{"}$

$\displaystyle = \lim_{x\to\infty} \frac{0 - (1\cdot \ln x + x \cdot \frac{1}{x})}{e^x} = \lim_{x\to\infty} \frac{-\ln x - 1}{e^x} = \text{"}\frac{-\infty}{\infty}\text{"}$

$\displaystyle = \lim_{x\to\infty} \frac{-1/x}{e^x} = \underline{\underline{0}}$ D

05/2017, 14:



$\displaystyle \lim_{x\to 0^+} x\cdot \ln x =$ "$0\cdot(-\infty)$"

$\displaystyle = \lim_{x\to 0^+} \frac{\cancel{x}\cdot \ln x}{1} \cdot \frac{\cancel{1/x}}{1/x} = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \text{"}\frac{-\infty}{\infty}\text{"}$

$\displaystyle = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} \cdot \frac{x^2}{x^2} = \lim_{x\to 0^+} \frac{x}{-1} = \underline{\underline{0}}$ B

15.

$$f(x) = 1 + 8x - 2x^2 - x^4$$

$$f'(x) = 8 - 4x - 4x^3$$

$$= -4(x^3 + x - 2)$$

$$= -4 \cdot (x-1)(x^2 + x + 2)$$

$$x^3 + x - 2 = 0$$

Mulige heltallsløsn:

$$x = \pm 1, \pm 2$$

$x = 1$ er løsn

$(x-1)$ faktor.

$$(x^3 + x - 2) : (x - 1) = x^2 + x + 2$$

$$-(x^3 - x^2)$$

$$x^2 + x - 2$$

$$-(x^2 - x)$$

$$2x - 2$$

$$2x - 2$$

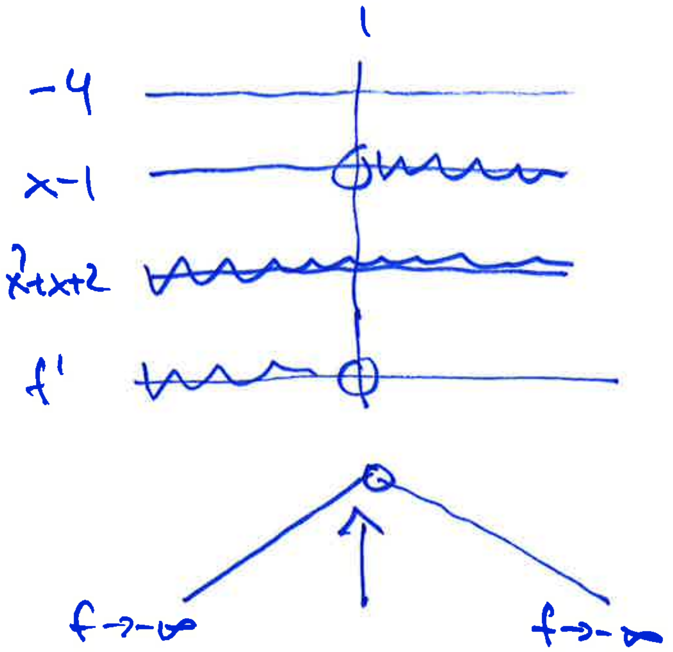
$$0$$

$$x^2 + x + 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1-8}}{2}$$

ingen løsn.

irredusibelt kvadratisk faktor



$x = 1$

lokalt maks

$\Rightarrow$ globalt maks

ingen lokale min

$\Rightarrow$ ingen globale min (B)

05/2016, L4:

$$f(x) = (x^2-6x-15)e^x = u \cdot v$$

$$f'(x) = (2x-6)\cdot e^x + (x^2-6x-15)\cdot e^x$$

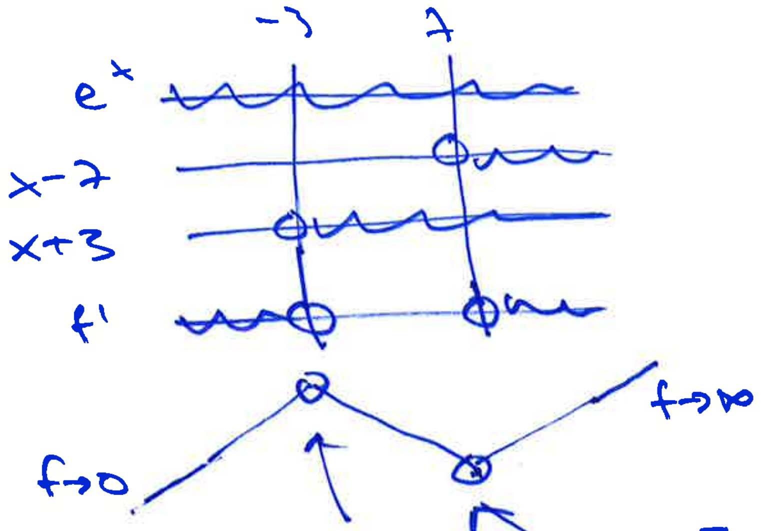
$$= \underline{e^x \cdot (x^2-4x-21)} \leftarrow$$

$$= e^x(x-7)(x+3)$$

$$x^2-4x-21=0$$

$$x = \frac{4 \pm \sqrt{16+84}}{2}$$

$$= \frac{4 \pm 10}{2} = 7, -3$$



$$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} \frac{x^2-6x-15}{e^{-x}}$$

$$= \frac{\infty}{\infty} = \underline{0}$$

$x=-3$
lokalt
maks

$f(-3) = 12e^{-3}$

ikke globalt
maks

$x=7$
lokalt
min

$f(7) = -8e^7$

globalt min

siden $-8e^7 < 0$

11/2015, Oppg 14

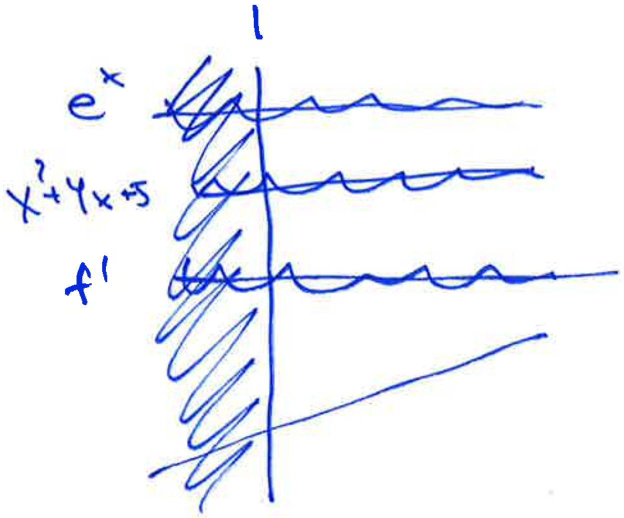
$f(x) = (x^2+2x+3)e^x$

$f'(x) = (2x+2)e^x + (x^2+2x+3)\cdot e^x$

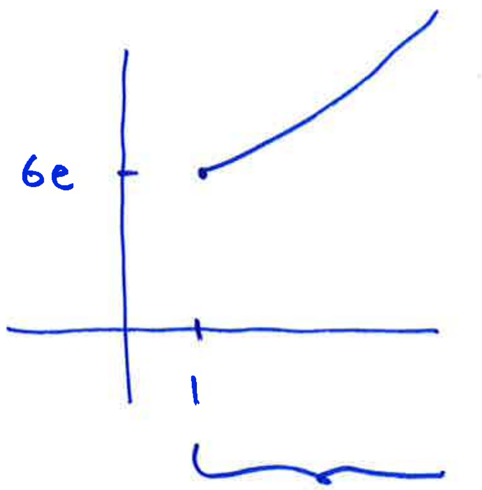
$= e^x \cdot (x^2+4x+5)$

$x^2+4x+5=0$

$x = \frac{-4 \pm \sqrt{16-4\cdot 5}}{}$



$V_f =$
$[6e, \infty)$



$D_f = [1, \infty)$

$D_{f^{-1}} = V_f$

$= [6e, \rightarrow)$

c