Forelesning 12

MET1180 BI

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Matematikk

Plan:

1. Eksponensial- og logaritmefunksjoner
2. Derivasjon

Pensum:

[E] 3.12–3.13, 4.1–4.3

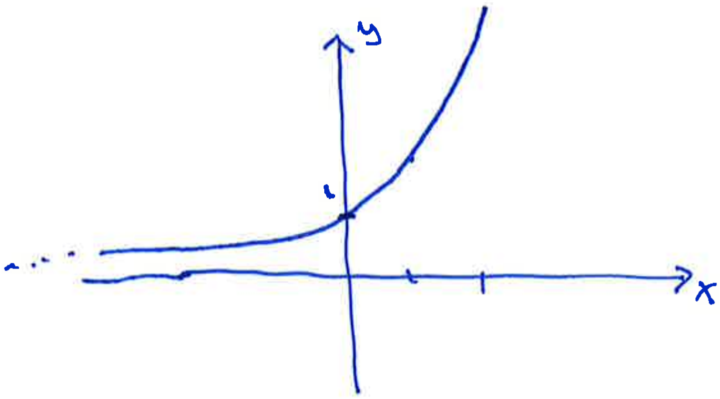
1 Eksponensial funksjoner

$f(x) = a^x$, $a > 0$ $(a \neq 1)$ først fall

Eks: $f(x) = 2^x$

x	0	1	2	3
$f(x)$	1	2	4	8

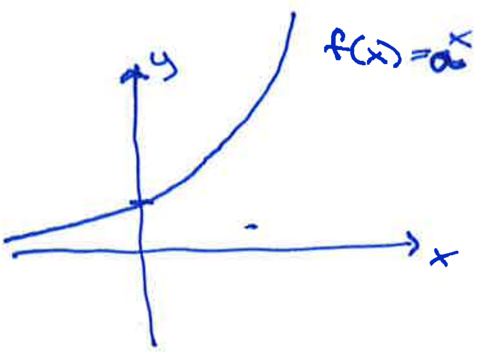
	-1	-2
	$1/2$	$1/4$



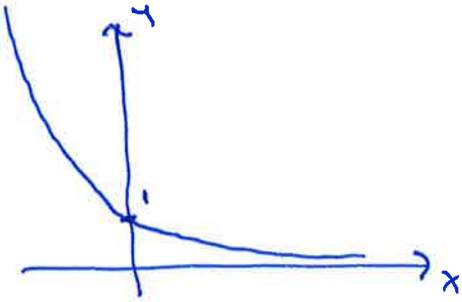
Merk:

- $f(x) = a^x$ er definert for alle x
- $f(x) = a^x$ er voksende for $a > 1$, avtagende for $0 < a < 1$
- $f(x) = a^x$ er kontinuerlig

1) $a^x \cdot a^y = a^{x+y}$
2) $a^x : a^y = a^{x-y}$
3) $(a^x)^y = a^{x \cdot y}$



$a>1$



$0<a<1$

Viktig eksempel: $\boxed{f(x)=e^x}$, $e = 2{,}71828\ldots$ Kalk: $\boxed{e^x}$

Eks: Eiendom har verdi 100 mill. kr. Verdien øker med 7% i året.
Verdi etter x år:

$V_1(x) = 100 \cdot 1{,}07^x$ eller $V_2(x) = 100 \cdot \left(e^{0{,}07}\right)^x = 100 \cdot e^{0{,}07x}$
(Kont.)

Eks: Anta $V_2(x) = 100 \cdot e^{0{,}07x}$
Når er verdien økt til 350 mill. kr?

$$100 \cdot e^{0{,}07x} = 350 \quad | :100$$

$$e^{0{,}07x} = \frac{350}{100} = 3{,}5$$

Må bruke logaritmer

$$e^{0{,}07x} = 3{,}5 \quad | \ln(-)$$

$$\ln\left(e^{0{,}07x}\right) = \ln(3{,}5)$$

$$\frac{0{,}07x}{0{,}07} = \ln(3{,}5) \simeq \frac{1{,}2528}{0{,}07}$$

$$x \simeq 18 \text{ år}$$

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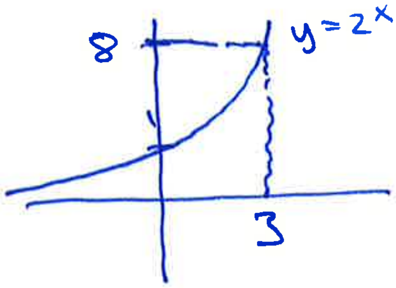
Logaritme funksjoner

$f(x) = \log_a(x)$ er den omvendte funksjonen til a^x, for $a>0$, $a \neq 1$.

Eks: $\log_2(8) = 3$ fordi



Generelt: $\log_a(x) =$ det tallet y slik at $a^y = x$

Vanlige logaritmer: $\ln(x) = \log_e(x)$: den naturlige logaritmen

$\left(\log(x) = \log_{10}(x) \right.$: den Briggske logaritmen $\left. \right)$

På kalk: $\boxed{\ln(x)}$

Regneregler:

i) $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

ii) $\log_a(x/y) = \log_a(x) - \log_a(y)$

iii) $\log_a(x^n) = n \cdot \log_a(x)$

$a^{m+n} = a^m \cdot a^n$

$a^{m-n} = a^m / a^n$

spesialtilfelle $a=e$

i) $\ln(x \cdot y) = \ln(x) + \ln(y)$

ii) $\ln(x/y) = \ln(x) - \ln(y)$

iii) $\ln(x^n) = n \cdot \ln(x)$

Eks: $2^x = 7$

$\swarrow \log_2(-)$ $\qquad$ $\searrow \ln(-)$

$$\log_2(2^x) = \log_2(7)$$
$$x = \log_2(7)$$

$$\ln(2^x) = \ln(7)$$
$$\frac{x \cdot \cancel{\ln(2)}}{\cancel{\ln(2)}} = \frac{\ln(7)}{\ln(2)}$$
$$x = \frac{\ln(7)}{\ln(2)} \simeq \underline{2.8}$$

$$\boxed{\text{Regneregler:} \quad \log_a(x) = \frac{\ln(x)}{\ln(a)}}$$

a^x og $\log_a(x)$ er omvendte funksjoner:

$$\log_a(a^x) = x$$
$$a^{\log_a(x)} = x$$

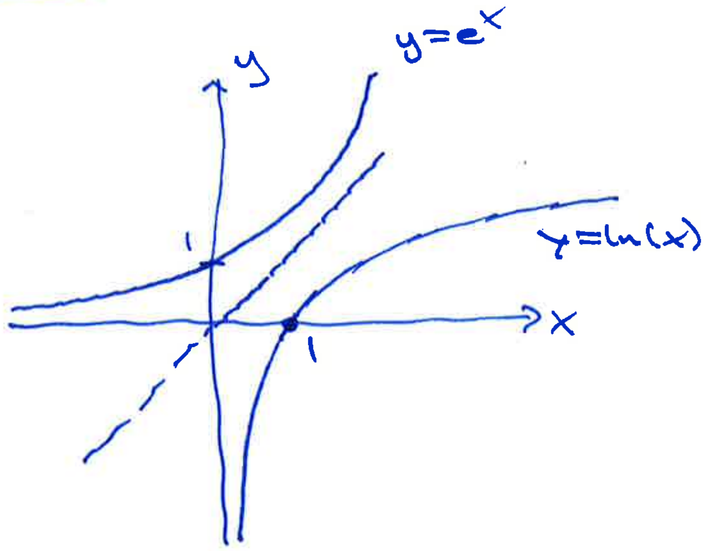
Eks: $2^{x+1} = 3^x \quad | \ln(-)$

$$\ln(2^{x+1}) = \ln(3^x)$$
$$(x+1)\ln(2) = x \cdot \ln(3)$$
$$x \cdot \ln(2) - x \cdot \ln(3) = -\ln(2)$$
$$\frac{x \cdot \cancel{(\ln(2) - \ln(3))}}{\cancel{\ln(2) - \ln(3)}} = \frac{-\ln(2)}{\ln(2) - \ln 3}$$
$$x = \frac{-\ln(2)}{\ln(2) - \ln(3)} \cdot \frac{(-1)}{(-1)} = \underline{\underline{\frac{\ln(2)}{\ln(3) - \ln(2)}}}$$

Eks: $\log_5(x) = 2 \quad | 5^{-}$

$$5^{\log_5(x)} = 5^2$$
$$x = \underline{\underline{25}}$$

Funksjonen $f(x)=\ln(x)$:



x	1	2	3
$\ln x = f(x)$	0	$\ln(2) \approx 0.69$	$\ln(3) \approx \ldots$

* $D_f = (0, \rightarrow)$: $\ln(x)$ er definert for $\underline{x>0}$
* $f(x)=\ln(x)$ er voksende

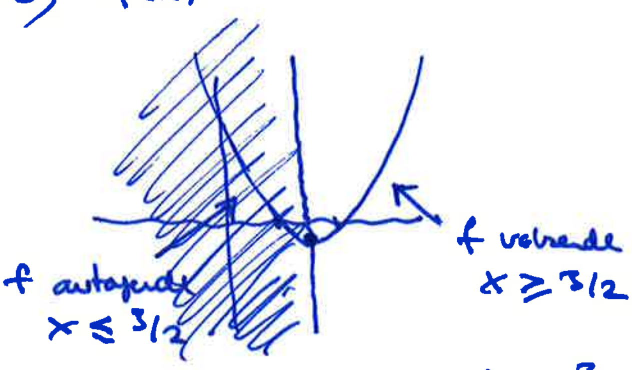
<u>Merk:</u> e^x vokser mye raskere enn $y=x$,
$\ln(x)$ vokser mye saktere enn $y=x$

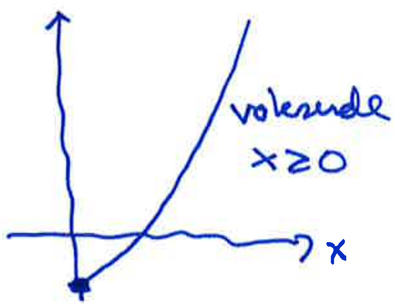
Oppgaver fra læreboken:

<u>3.11.2.</u> Finn D_f slik at f^{-1} eksisterer når:

b) $f(x)=x^2-3x+2$

c) $f(x)=x\sqrt{x}-1$





Symmetri-linje: $x = \frac{-b}{2a} = \frac{3}{2}$

$f(3/2) = -\frac{1}{4}$

$D_f = [3/2, \rightarrow)$

$\Rightarrow f^{-1}$ fins.

$D_f = [0, \rightarrow)$

$\Rightarrow f^{-1}$ fins.

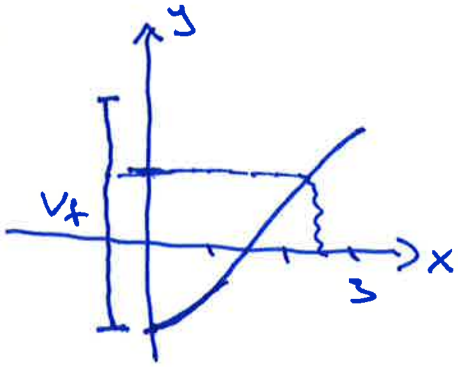
3.11.3. $f(x) = \ldots$ kontinuerlig funksjon

$D_f = [0,3]$

Finn f^{-1} når: a) f voksende på $[0,3]$

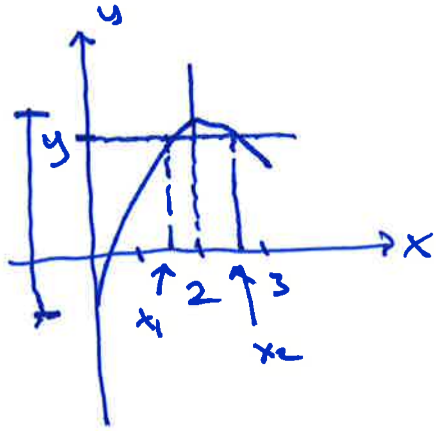
b) f voksende på $[0,2]$ og avtagende på $[2,3]$

a)



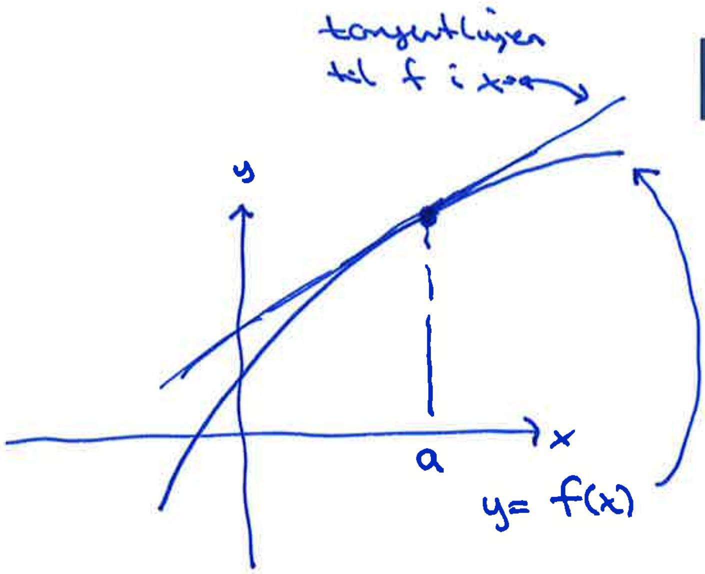
f^{-1} fins siden f er voksende

b)



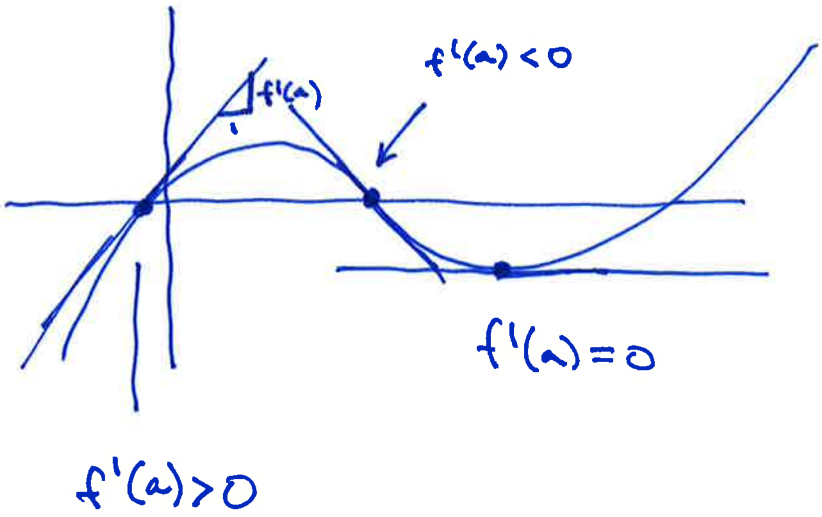
f^{-1} <u>fins ikke</u> siden $f(x_1) = f(x_2)$ med $x_1 \neq x_2$.

② Derivasjon



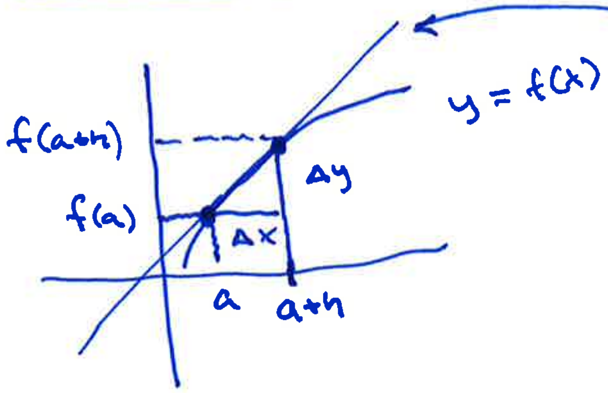
Den deriverte:

$f'(a)$ = stigningstallet
til tangentlinjen
til $y=f(x)$ i $x=a$



$f'(a)$: den deriverte
til f i $x=a$
= et tall

Finne $f'(a)$ vha sekanter:

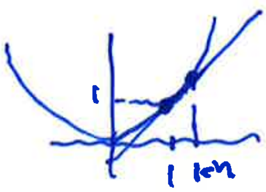


sekant: rett linje gjennom
to pkt. på grafen.

Stigningstallet til sekanten:

$$\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{(a+h)-a} = \frac{f(a+h)-f(a)}{h}$$

Eks: $f(x) = x^2$
$a = 1 \quad a+h = 1+h$



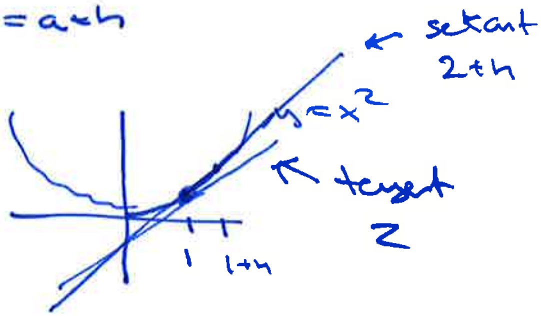
$$\frac{\Delta y}{\Delta x} = \frac{f(1+h)-f(1)}{h} = \frac{(1+h)^2-1}{h}$$

$$= \frac{\cancel{1}+2h+h^2-\cancel{1}}{h} = \underline{2+h}$$

Definisjon:

$$f'(a) = \underbrace{\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}}$$

Stigningstallet
til sekanten til
f grafen
$x=a$, $x=a+h$



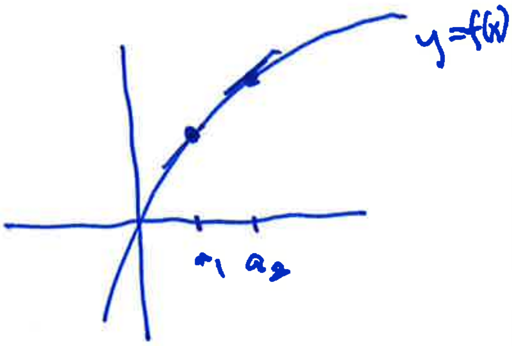
Eks: $f(x) = x^2$

$$f'(1) = \lim_{h\to 0} (2+h)$$

$$= \underline{\underline{2}}$$

Den deriverte funksjonen:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$



Eks: $f(x) = x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} (2x+h) = \underline{\underline{2x}}$$

$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h}$$

$$= \frac{\cancel{x^2} + 2hx + h^2 - \cancel{x^2}}{h}$$

$$= \underline{2x+h}$$

$\underline{f'(x) = 2x}$: $\left\{ \begin{array}{l} f'(0) = 0 \\ f'(1) = 2 \\ \vdots \end{array} \right.$

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Regneregler:

① $f(x) = x^n \Rightarrow f'(x) = \underline{n \cdot x^{n-1}}$

(n et tall)

Eks: $(x^2)' = 2x^1 = \underline{2x}$

$(x^5)' = \underline{5x^4}$

$(x^{-1})' = -1 \cdot x^{-2}$ dvs: $\left(\frac{1}{x}\right)' = \underline{\underline{-\frac{1}{x^2}}}$

$(x^{1/2})' = \frac{1}{2} \cdot x^{-1/2}$ " $(\sqrt{x})' = \frac{1}{2} \cdot \frac{1}{\sqrt{x}}$

$= \underline{\underline{\frac{1}{2\sqrt{x}}}}$

② $\left.\begin{aligned}(u+v)' &= u' + v' \\ (u-v)' &= u' - v' \\ (c \cdot u)' &= c \cdot u'\end{aligned}\right\}$ $\left.\begin{aligned}u &= u(x) \\ v &= v(x)\end{aligned}\right\}$ uttrykk i x

c: konstant

Eks: $(x^2 + 2x - 3)' = (x^2)' + (2x)' - (3)'$

$= 2x + 2 \cdot 1 - 0$

$= \underline{\underline{2x + 2}}$

Dette betyr: $\left\{\begin{aligned}(u(x) + v(x))' &= u'(x) + v'(x) \\ (u(x) - v(x))' &= u'(x) - v'(x) \\ (c \cdot u(x))' &= c \cdot u'(x)\end{aligned}\right.$

Skrivemåte: Den deriverte funksjonen til $f(x)$ kan skrives:

$f'(x) = (f(x))' = \frac{df}{dx} \leftarrow$ Leibniz' skrivemåte