FORELESNING 17

MET1180 BI

EIVIND ERIKSEN, JAN 24 2018

MATEMATIKK

Plan:

1. Integrasjon av rasjonale uttrykk
2. Arealberegning og anvendelser. Bestemte integral.

Pensum:

[E] 5.5 - 5.7

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1. Integrasjon av rasjonale uttrykk

- delbrøksoppspalting
- polynomdivisjon

Eks:

$$\int \frac{x}{x-1}\,dx =$$

$$= \int \left(1 + \frac{1}{x-1}\right) dx$$

$$= x + \int \frac{1}{x-1}\,dx$$

$$= \underline{\underline{x + \ln|x-1| + C}}$$

$$x : x-1 = 1$$
$$-(x-1)$$
$$\overline{\quad 1 \quad}$$

$$\frac{x}{x-1} = 1 + \frac{1}{x-1}$$

$$\boxed{\begin{aligned} u &= x-1 \\ du &= 1\cdot dx \end{aligned}}$$

$$\int \frac{1}{x-1}\,dx = \int \frac{1}{u}\cdot du$$
$$= \ln|u| + C$$
$$= \ln|x-1| + C$$

Husk: $\int \frac{A}{ax+b}\,dx = \frac{A}{a}\cdot\ln|ax+b| + C$

Eks: $$\int \frac{1}{x^2-3x+2}\,dx = \int \frac{1}{x-2} + \frac{-1}{x-1}\,dx$$

delbrøks-
oppspalting

$$\frac{1}{x^2-3x+2} = \frac{1}{(x-2)\cdot(x-1)} = \frac{A}{x-2} + \frac{B}{x-1} \quad \Big|\cdot (x-2)(x-1)$$

$$1 = A\cdot(x-1) + B\cdot(x-2)$$

$x=1$: $1 = A\cdot 0 + B\cdot(-1)$

$\underline{B=-1}$

$x=2$: $1 = A\cdot 1 + B\cdot 0$

$\underline{A=1}$

$1 = \underline{A}x - A + \underline{B}x - 2B$

$1 = \underset{\overset{\shortparallel}{0}}{(A+B)}x \underbrace{-A-2B}_{=1}$

$A+B=0$ $\qquad$ $B=-A$ $\qquad$ $\underline{B=-1}$

$-A-2B=1$ $\qquad$ $-A-2\cdot(-A)=1$

$\underline{A=1}$

$$= \int \frac{1}{x-2}\,dx + \int \frac{-1}{x-1}\,dx = \underline{\underline{\ln|x-2| - \ln|x-1| + C}}$$

Eks: $$\int \frac{x}{x^2-4x+4}\,dx =$$

$$= \int \frac{1}{x-2} + \frac{2}{(x-2)^2}\,dx$$

$$= \underline{\underline{\ln|x-2| - \frac{2}{x-2} + C}}$$

$$\int \frac{2}{(x-2)^2}\,dx = \int \frac{2}{u^2}\,du = 2\cdot\int u^{-2}\,du$$

$\boxed{u=x-2 \quad du=dx}$

$$= 2\cdot\left(\frac{1}{-1}\cdot u^{-1}\right) + C$$

$$= -2\cdot\frac{1}{x-2} + C$$

$$\frac{x}{x^2-4x+4} = \frac{x}{(x-2)^2}$$

$$\frac{x}{(x-2)^2} = \frac{A}{(x-2)} + \frac{B}{(x-2)^2}$$

$x = A\cdot(x-2) + B\cdot 1$

$x = \underset{\overset{\shortparallel}{1}}{A}x \underbrace{-2A+B}_{\overset{\shortparallel}{0}}$

$\underline{A=1}$

$-2A+B=0$ $\qquad$ $\underline{B=2}$

Eks: $\displaystyle\int \frac{3}{x^2-4x+7}\,dx = \underline{\text{noe med } \arctan(u)}$

$$\frac{3}{x^2-4x+7} =$$

$x^2-4x+7=0$

$x = \dfrac{4 \pm \sqrt{16-28}}{2}$

ingen løsn.

$\Downarrow$

ingen faktorisering

Formel: $\boxed{\displaystyle\int \frac{1}{x^2+1}\,dx = \arctan(x) + C}$

$\left(\tan x = \dfrac{\sin x}{\cos x}, \quad \arctan x = \right.$ omvendt fn. til $\left.\tan(x) \right)$

Eks: $\displaystyle\int \frac{3}{x^2-4x+7}\,dx = \int \frac{1}{u^2+1}\,\frac{du}{1/\sqrt{3}} \overset{\cdot\sqrt{3}}{\underset{\cdot\sqrt{3}}{=}} \int \frac{\sqrt{3}}{u^2+1}\,du$

$$\frac{3}{x^2-4x+7} = \frac{3}{(x-2)^2+3} \cdot \frac{\frac{1}{3}}{\frac{1}{3}} = \frac{1}{\frac{(x-2)^2}{3}+1} = \frac{1}{u^2+1} \quad \left(u = \frac{x-2}{\sqrt{3}} \right)$$

Derfor:

$$\boxed{\begin{aligned} u &= \frac{x-2}{\sqrt{3}} \\ du &= \frac{1}{\sqrt{3}} \cdot dx \end{aligned}}$$

$$= \sqrt{3} \cdot \arctan(u) + C = \underline{\underline{\sqrt{3}\,\arctan\left(\frac{x-2}{\sqrt{3}}\right) + C}}$$

② Arealberegning og anvendelser

Bestemt integral:

Eks:

$$\int_0^1 2x\,dx = \left[x^2+C\right]_0^1$$

uavhengig av C

$$= (1^2+C)-(0^2+C)$$

$$= 1+C-0-C = \underline{\underline{1}}$$

Generelt:

$$\int_a^b f(x)\,dx = \left[F(x)+C\right]_a^b = F(b)-F(a)$$

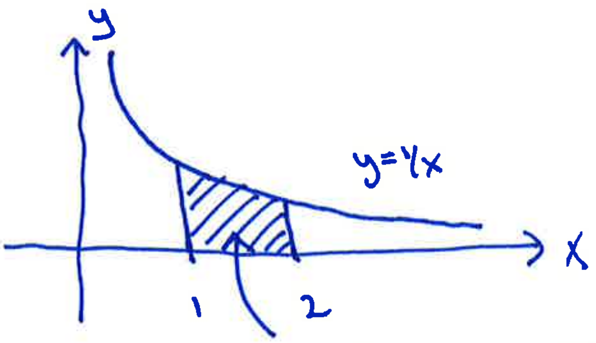
der $F(x)$ er en antiderivert til $f(x)$,

dvs $F'(x)=f(x)$.

Eks:

$$\int_1^2 \frac{1}{x}\,dx = \left[\ln|x|\right]_1^2 = \ln(2)-\ln(1)$$

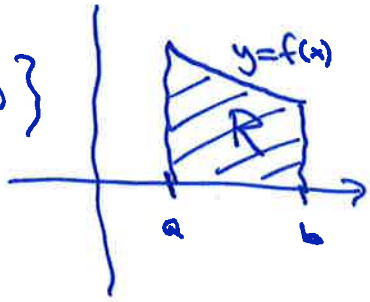
$$= \underline{\underline{\ln(2)}} \simeq 0.693$$



areal $= \ln(2) \simeq 0.693$

Resultat:

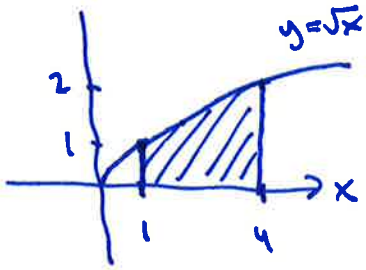
Hvis $f(x)$ er en kontinuerlig funksjon med $f(x) \geq 0$ for alle x i $[a,b]$, da er arealet av $R = \{(x,y) : a \leq x \leq b,\ 0 \leq y \leq f(x)\}$



gitt ved

$$A(R) = \int_a^b f(x)\,dx$$

Eks: R er området under $f(x) = \sqrt{x}$ når x er i $[1,4]$

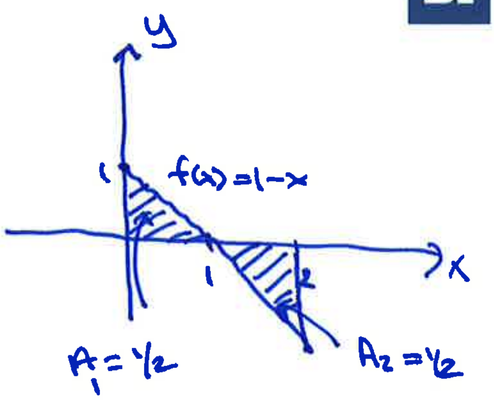


$$A(R) = \int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2}\,dx$$

$$= \left[\frac{2}{3} \cdot x^{3/2}\right]_1^4 = \left[\frac{2}{3}\, x \cdot \sqrt{x}\,\right]_1^4$$

$$= \frac{2}{3} \cdot 4 \cdot 2 - \frac{2}{3} \cdot 1 \cdot 1 = \frac{16}{3} - \frac{2}{3} = \underline{\underline{\frac{14}{3}}} \approx 4{,}67$$

Eks: $\displaystyle\int_0^2 1-x \, dx$



$$= \left[x - \tfrac{1}{2}x^2\right]_0^2$$

$$= \left(2 - \tfrac{1}{2}\cdot 4\right) - (0) = \underline{\underline{0}}$$

$$= +A_1 - A_2 = 1/2 - 1/2 = 0$$

$$A_1 + A_2 = 1/2 + 1/2 = \underline{\underline{1}}$$

Resultat:

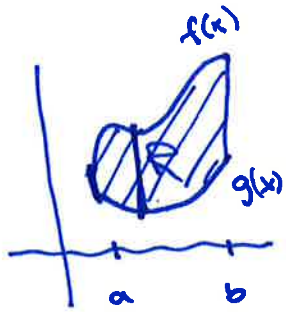
Hvis $\left\{\begin{matrix} f(x) \\ g(x) \end{matrix}\right\}$ er kont. på $[a,b]$ og $f(x) \geq g(x)$

for alle x i $[a,b]$, da er arealet

av $R = \{(x,y) : a \leq x \leq b,\ g(x) \leq y \leq f(x)\}$

gitt ved

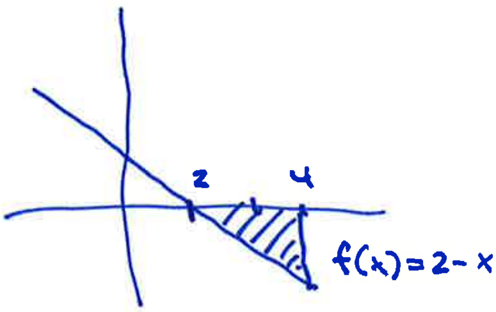
$$A(R) = \int_a^b \left(f(x) - g(x)\right) dx$$



Eks:

$$A = \int_2^4 \left(0 - f(x)\right) dx = \int_2^2 - f(x)\, dx$$

$$= - \int_2^4 f(x)\, dx = \underline{\underline{2}}$$



$$\int_2^4 f(x)\, dx = -A$$

$$\text{''}$$

$$\int_2^4 2-x \, dx = \left[2x - \tfrac{1}{2}x^2\right]_2^4 = (8-8) - (4-2) = -2$$

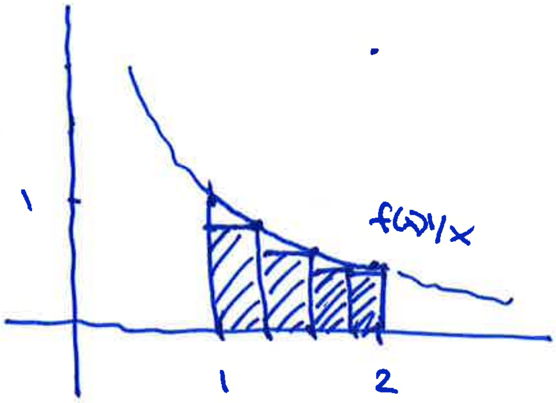
Riemann-summer:

Eks:

$$\int_1^2 \frac{1}{x}\,dx$$

$$= \left[\ln x\right]_1^2$$

$$= \ln(2) \simeq 0{,}693$$



n = antall delintervall = 4

Riemannsum:

god tilnærming til arealet under $1/x$ mellom $x=1$ og $x=2$

$$\left\{ \begin{aligned} &0.25\cdot\frac{1}{1{,}25} + 0{,}25\cdot\frac{1}{1{,}5} \\ &+ 0{,}25\cdot\frac{1}{1{,}75} + 0{,}25\cdot\frac{1}{2} \end{aligned} \right.$$

tilnærmingen blir bedre og bedre jo større n = antall delintervall er

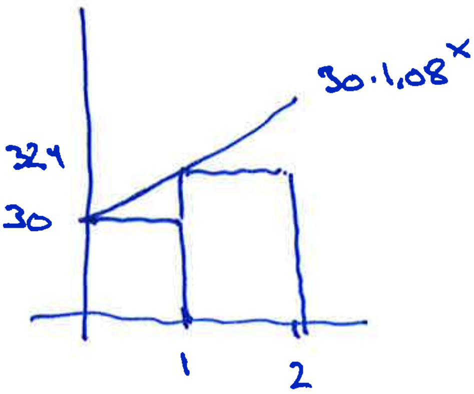
Bestemt integral = en måte å summere opp størrelser som endrer seg på en kontinuerlig måte

Eks: Utleie av eiendom

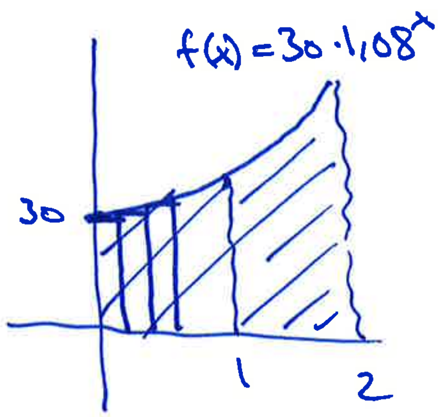
leie nå: 30 mill kr

økning: 8% per år.

Leie neste 2 år: ?



Sum: 30 + 32.4 = 62.4 mill. kr



$$(a^x)' = a^x \cdot \ln(a)$$

$$\int a^x \, dx = a^x \cdot \frac{1}{\ln(a)} + C$$

$$\int_0^2 30\cdot 1{,}08^x \, dx$$

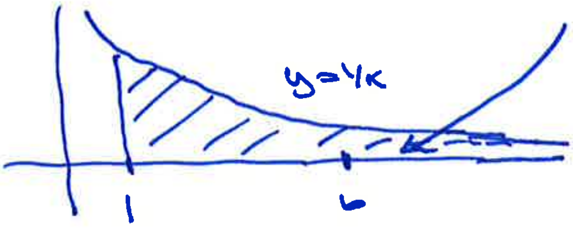
$$= \left[30 \cdot a^x \cdot \frac{1}{\ln(a)} \right]_0^2$$

$$= \frac{30}{\ln(1{,}08)} \cdot 1{,}08^2 - \frac{30}{\ln(1{,}08)} \cdot 1$$

$$= \frac{30 \cdot (1{,}08^2 - 1)}{\ln(1{,}08)}$$

$$\approx 64.86 \text{ mill. kr}$$

Eks:



$$R = \{(x,y) : x \geq 1 , 0 \leq y \leq 1/x\}$$

$$A(R) = \int_1^\infty \frac{1}{x}\,dx = \lim_{b\to\infty} \int_1^b \frac{1}{x}\,dx$$

$$= \Big[\ln x\Big]_1^\infty = \lim_{b\to\infty} \Big[\ln x\Big]_1^b$$

$$= \ln(\infty) - \ln(1) \qquad \lim_{b\to\infty} (\underbrace{\ln b}_{\downarrow\ \infty} - \underbrace{\ln 1}_{=0}) = \infty$$

$$= \infty$$

Eks:

$$\int_1^\infty \frac{1}{x^2}\,dx = \Big[-\frac{1}{x}\Big]_1^\infty$$

$$= \lim_{b\to\infty} \left(-\frac{1}{b}\right) - \left(-\frac{1}{1}\right)$$

$$= \lim_{b\to\infty} \left(\underbrace{-\frac{1}{b}}_{\downarrow\ 0} + \underbrace{1}_{=1}\right) = \underline{\underline{1}}$$