FORELESNING 20

MET 1180 — MATEMATIKK

EIVIND ERIKSEN, JAN 31 2018

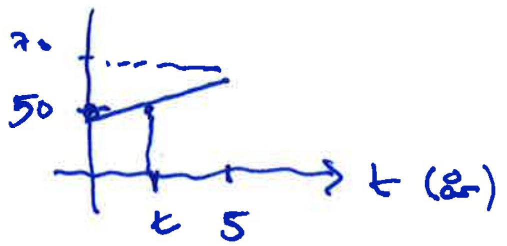
Plan:

1. Eksamensoppgaver i integrasjon
   MET11803: 12/2015 Oppg 2, 06/2016 Oppg. 3
2. Likningssystemer i flere variabler

Pensum:

[E] 6.1

(1) Oppgave 5.7.1:



$$f(t) = \alpha + \beta t$$
$$= 50 + 4t$$

a) $r = 0\%$:

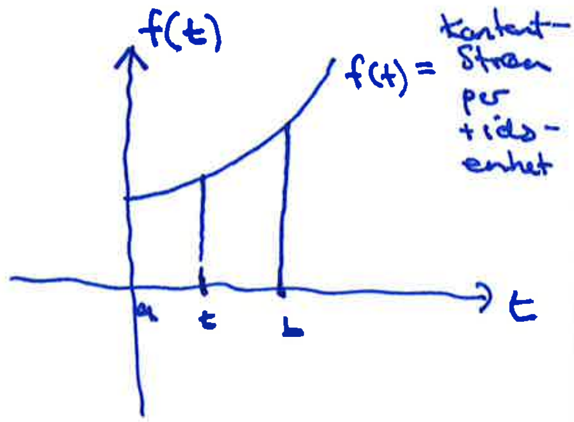
$$\int_0^5 (50+4t)e^{-0\cdot t}\, dt$$

$$= \int_0^5 50 + 4t \, dt$$

$$= \left[50t + 2t^2\right]_0^5$$

$$= (50\cdot 5 + 2\cdot 5^2) - 0$$

$$= 250 + 50 = \underline{\underline{300}}$$



Total kontantstrøm:

$$\int_a^b f(t)\, dt$$

Nåverdi:

$$\int_{a=0}^b f(t)\cdot e^{-rt}\, dt$$

b) $\underline{r = 5\% = 0{,}05}$:

$$\int_0^5 (50+4t)\cdot e^{-0{,}05t}\,dt$$

$$= \int_0^5 50\cdot e^{-0{,}05t}\,dt + \int_0^5 4te^{-0{,}05t}\,dt$$

$$= \left[50\left(\frac{1}{-0{,}05}\right)e^{-0{,}05t}\right]_0^5$$

$$+ \left[4\left[\left(-\frac{1}{r}\right)te^{-rt} - \frac{1}{r^2}e^{-rt}\right]\right]_0^5$$

$$\int \overset{v}{t}\,\overset{u'}{e^{-rt}}\,dt =$$

$$= \frac{1}{-r}te^{rt} + \int \frac{1}{r}e^{-rt}\cdot 1\,dt$$

$$= \frac{1}{-r}te^{rt} - \frac{1}{r}\cdot\frac{1}{r}e^{-rt} + C$$

$u = \frac{1}{r}e^{rt}$ $\quad v = t$

$u' = e^{rt}$ $\quad v' = 1$

$$= -\frac{50}{0{,}05}\left(e^{-0{,}25} - 1\right) - \frac{4}{0{,}05}\left(5e^{-0{,}25} - 0\right)$$

$$- \frac{4}{0{,}05^2}\left(e^{-0{,}25} - 1\right)$$

$$= -1000\left(e^{-0{,}25}-1\right) - 80\left(5e^{-0{,}25}\right) - 1600\left(e^{-0{,}25}-1\right)$$

$$= \underline{\underline{2600 - 3000e^{-0{,}25}}} \simeq 263{,}6 \text{ mill kr}$$

5.7.2. $\quad f(t) = (5+t)\cdot e^{\sqrt{t}}$

Total innleht: $\displaystyle\int_0^{16} (5+t)e^{\sqrt{t}}\,dt = \int_0^4 (5+t)\cdot e^u\cdot 2\sqrt{t}\,du$

$$\boxed{\begin{aligned} u &= \sqrt{t} \\ du &= \frac{1}{2\sqrt{t}}\,dt \end{aligned}}$$

$t=0:\ u=0$

$t=16:\ u=4$

$$= \int_0^4 (5+u^2)\cdot 2u\,e^u\,du$$

$$= \int_0^4 (2u^3+10u)\,e^u\,du$$

$$= \left[(2u^3+10u)\cdot e^u - \int (6u^2+10)\,e^u\,du\right]_0^4$$

$$= \left[(2u^3+10u)e^u\right]_0^4 - \int_0^4 (6u^2+10)e^u\,du$$

$$= \text{———} \;\;{''}\;\; \text{———} - \left((6u^2+10)e^u - \int 12u\,e^u\,du\right)_0^4$$

$$= \left[(2u^3+10u)e^u - (6u^2+10)e^u\right]_0^4 + \int_0^4 12u\,e^u\,du$$

$$= \text{———} \;\;{''}\;\; \text{———} + \left[12u\,e^u - \int 12e^u\,du\right]_0^4$$

$$= \left[(2u^3+10u)e^u - (6u^2+10)e^u + 12u\,e^u - 12e^u\right]_0^4$$

$$= \left(168e^4 - 106e^4 + 48e^4 - 12e^4\right) - (-10-12)$$

$$= \underline{\underline{22+98e^4}} \simeq 5.372,6 \text{ mill. kr}$$

Opps. 5.4.4

$$\int u'v\,dx = uv - \int uv'\,dx$$

BI

a)

$$\int \frac{\ln x}{x}\,dx = \int \underbrace{\frac{1}{x}}_{u'} \cdot \underbrace{\ln x}_{v}\,dx = (\ln x)^2 - \int \ln x \cdot \frac{1}{x}\,dx$$

$$\boxed{\begin{matrix} u = \ln x & v = \ln x \\ u' = 1/x & v' = 1/x \end{matrix}}$$

$$\int \frac{1}{x}\cdot \ln x\,dx = (\ln x)^2 - \int \frac{1}{x}\cdot \ln x\,dx$$

$$I = (\ln x)^2 - I$$

$$2I = (\ln x)^2 + C$$

$$I = \frac{(\ln x)^2}{2} + C$$

$$\int \frac{\ln x}{x}\,dx = I = \underline{\underline{\frac{(\ln x)^2}{2} + C}}$$

b)

$$\int \underbrace{x^2}_{v} \underbrace{e^x}_{u'}\,dx = x^2 e^x - \int 2x e^x\,dx$$

$$\boxed{\begin{matrix} u = e^x & v = x^2 \\ u' = e^x & v' = 2x \end{matrix}}$$

$$= x^2 e^x - \int \underbrace{2x}_{v} \underbrace{e^x}_{u'}\,dx =$$

$$\boxed{\begin{matrix} u = e^x & v = 2x \\ u' = e^x & v' = 2 \end{matrix}}$$

$$= x^2 e^x - \left(2x e^x - \int 2e^x dx\right) = \underline{\underline{x^2 e^x - 2x e^x + 2e^x + C}}$$

Eksamen 06/2016. Opps 3:

a) $$\int \frac{3x-4}{x^2+x}\,dx = \int\left(\frac{-4}{x} + \frac{7}{x+1}\right)dx = \underline{-4\cdot\ln|x| + 7\cdot\ln|x+1| + C}$$

Delbrøk:

$$\frac{3x-4}{x\cdot(x+1)} = \frac{\overset{-4}{A}}{x} + \frac{\overset{7}{B}}{x+1} \quad \Big|\cdot x(x+1)$$

$$3x-4 = A\cdot(x+1) + B\cdot x$$

$x=0$: $\quad -4 = A\cdot 1 + B\cdot 0 = A \quad \rightarrow A=-4$

$x=-1$: $\quad -7 = A\cdot 0 + B\cdot(-1) = -B \quad\quad B=7$

b) $$\int \underbrace{18x^2}_{u'}\underbrace{\ln(x+1)}_{v}\,dx = 6x^3\cdot\ln(x+1) - \int 6x^3\cdot\frac{1}{x+1}\,dx$$

$u = 18\cdot\frac{1}{3}x^3 = 6x^3 \qquad v = \ln(x+1)$

$u' = 18x^2 \qquad v' = \frac{1}{x+1}$

$$\frac{6x^3}{x+1} = 6x^2-6x+6 + \frac{-6}{x+1}$$

$$6x^3 : (x+1) = 6x^2 - 6x + 6$$
$$-(6x^3+6x^2)$$
$$-6x^2$$
$$-(-6x^2-6x)$$
$$6x$$
$$-(6x+6)$$
$$-6$$

$$= 6x^3 \cdot \ln(x+1) - \int \frac{6x^3}{x+1}\,dx$$

$$= \text{— '' —} - \int 6x^2 - 6x + 6 - \frac{6}{x+1}\,dx$$

$$= 6x^3 \cdot \ln(x+1) - 2x^3 + 3x^2 - 6x + 6 \cdot \ln|x+1| + C$$

c) $\int e^{\sqrt{x}}\,dx = \int e^u \cdot 2\sqrt{x} \cdot du = \int e^u \cdot 2u\,du$

$$\boxed{\begin{array}{c} u = \sqrt{x} \\ du = \frac{1}{2\sqrt{x}} \cdot dx \end{array}}$$

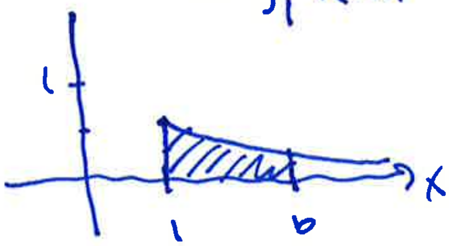
$$= \int \underbrace{2u}_{s}\,\underbrace{e^u}_{r'}\,du = 2ue^u - \int 2e^u\,du$$

$$= 2ue^u - 2e^u + C$$

$$= 2\sqrt{x}\,e^{\sqrt{x}} - 2e^{\sqrt{x}} + C$$

$$\boxed{\begin{array}{ll} r = e^u & s = 2u \\ r' = e^u & s' = 2 \end{array}}$$

d) $\lim_{b \to \infty} \int_1^b \frac{1}{x^2+x}\,dx = \int_1^\infty \frac{1}{x^2+x}\,dx$



arealet av R = området mellom $y = \frac{1}{x^2+x}$ og x-aksen når $x \geq 1$

$$\int \frac{1}{x^2+x}\,dx = \int \frac{1}{x} + \frac{-1}{x+1}\,dx = \ln|x| - \ln|x+1| + C$$

$$\frac{1}{x \cdot (x+1)} = \frac{A}{x} + \frac{B}{x+1} \quad \Big| \cdot (x+1)x$$

$$1 = A(x+1) + Bx$$

$x=0$: $1 = A$ $\quad A = 1$

$x=-1$: $1 = -B$ $\quad B = -1$

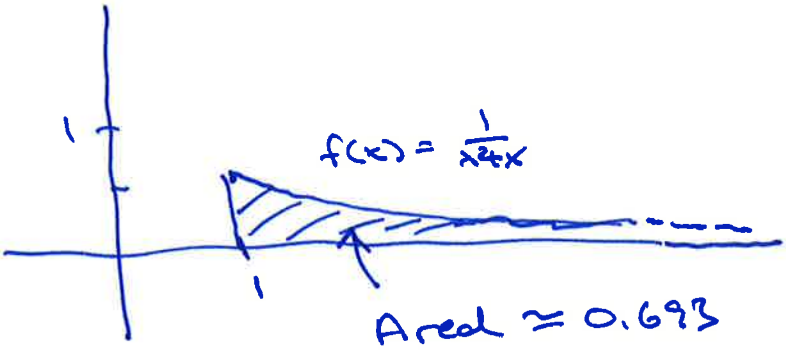
$$\lim_{b\to\infty}\int_1^b \frac{1}{x^2+x}\,dx = \lim_{b\to\infty}\Big[\ln|x| - \ln|x+1|\Big]_1^b$$

$$= \lim_{b\to\infty}\Big(\big[\ln(b)-\ln(b+1)\big] - \big[\ln 1 - \ln 2\big]\Big)$$

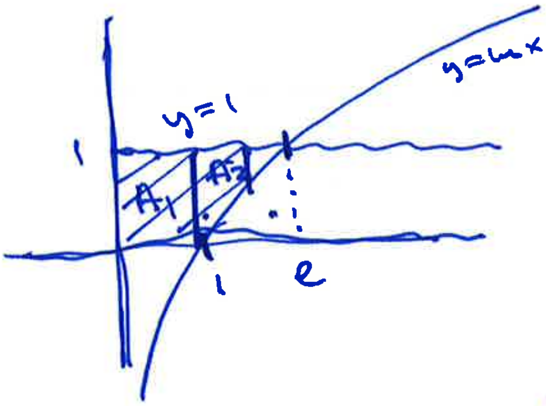
$$= \lim_{b\to\infty}\big(\underline{\ln(b)-\ln(b+1)} + \ln(2)\big)$$

$$= \ln(2) + \lim_{b\to\infty}\ln\Big(\underbrace{\frac{b}{b+1}}_{\to 1}\Big)$$

$$\int_1^\infty \frac{1}{x^2+x}\,dx = \underline{\underline{\ln(2)}} \simeq 0.693$$



Eksamen 12/2015, Oppg 2d):



Alt:

$$A_2 = (e-1)\cdot 1 - \int_1^e \ln x \, dx$$

$y = \ln x$:

$\ln x = 0$: $x = 1$

$e^{\ln x} = e^0$

$x = 1$

$\ln x = 1$:

$e^{\ln x} = e^1$

$x = e$

$$A = \underbrace{1\cdot 1}_{A_1} + \underbrace{\int_1^e 1 - \ln x \, dx}_{A_2}$$

$$= 1 + \int_1^e 1 \, dx - \int_1^e \ln x \, dx$$

$$= 1 + [x]_1^e - [x\cdot \ln x - x]_1^e$$

$$= 1 + (e-1) - \left[(e\cdot \ln e - e) - (1\cdot \ln 1 - 1)\right]$$

$$= e - (\cancel{e} - \cancel{e} + 1) = \underline{e - 1}$$

$$\simeq \underline{\underline{1{,}71}}$$

$\int \ln x \, dx$

$= \int 1\cdot \ln x \, dx$

$= x\cdot \ln x - \int 1 \, dx$

$u = x \quad v = \ln x$

$u' = 1 \quad v' = 1/x$

$= x \ln x - x + C$

② Likningssystemer

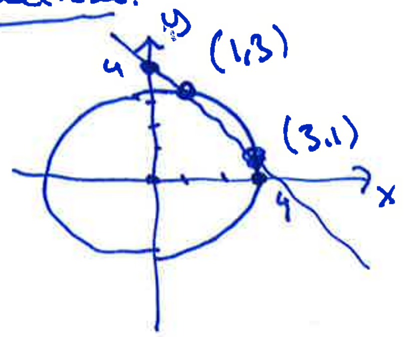
Eks: $x+y=4$

$x^2+y^2=10$

2x2 likn. system: 2 likninger i 2 ukjente

Definisjon: Lineært system: alle likninger i systemet er lineære

geometrisk:



① $x+y=4$

$y=4-x$

② $x^2+y^2=10$

sirkel med sentrum $(0,0)$ og radius $=\sqrt{10} \simeq 3.1$

$(x-x_0)^2+(y-y_0)^2=r^2$

Løsn. av likningssystemet: (x,y) som tilfredsstiller alle likn. i systemet

Innsettingsmetoden:

$x+y=4$

$x^2+y^2=10$

$\Rightarrow y=\underline{4-x}$

$\downarrow$

$x^2+(4-x)^2=10$

$\underline{x^2}+16-\underline{8x}+\underline{x^2}=10$

$2x^2-8x+6=0$

$x^2-4x+3=0$

$\underline{x=1}$ $\quad$ $\underline{x=3}$

$y=3$ $\quad$ $y=1$

Løsn: $(x,y)=(1,3)$, $(3,1)$

to løsninger