FORELESNING 26

MET 1180 BI

EIVIND ERIKSEN

MAR 07, 2018

MATEMATIKK

Plan:

1. Eksamen 12/2015 Opps. 3
2. Funksjoner i to variable

Pensum:

[E] 7.1 - 7.2

Innlevering: Frist fredag - epost eller papir

1 Eksamen 12/2015, Opps. 3

$$A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 2 \\ a \\ 2 \end{pmatrix} \qquad A \cdot \underline{x} = \underline{b}$$

a) $\det(A) = |A| = \boxed{a \cdot (a^2 - 4)} - 2 \cdot (2a-4) + 2 \cdot (4-2a)$

$= a^3 - 4a - 4a + 8 + 8 - 4a = \underline{\underline{a^3 - 12a + 16}}$

eller

$= a \cdot (a+2)(a-2) + (-4a + 8 + 8 - 4a)$

$= a(a+2)(a-2) + (-8a + 16)$

$= a(a+2)\underline{(a-2)} - 8(\underline{a-2})$

$= (a-2) \cdot [a(a+2) - 8]$

$= (a-2) \cdot (a^2 + 2a - 8)$

$= (a-2) \cdot (a-2)(a+4)$

$= \underline{\underline{(a-2)^2 \cdot (a+4)}}$

$a^2 + 2a - 8 = 0$

$a = \dfrac{-2 \pm \sqrt{4+32}}{2}$

$= \dfrac{-2 \pm 6}{2}$

$= 2, -4$

b) $\underline{a=0}$: $A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{pmatrix}$ $|A| = \underline{16}$ ($a=0$ i a))

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$$A^{-1} = \frac{1}{|A|} \cdot \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}^T = \frac{1}{16} \begin{pmatrix} -4 & 4 & 4 \\ 4 & -4 & 4 \\ 4 & 4 & -4 \end{pmatrix}^T$$

A symm.
⇓
(C_{ij}) symm.

$$= \frac{1}{16} \begin{pmatrix} -4 & 4 & 4 \\ 4 & -4 & 4 \\ 4 & 4 & -4 \end{pmatrix} = \frac{1}{4} \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$C_{11} = + \begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix} = -4$$

c) $A\underline{x} = \underline{b}$ $\Longleftrightarrow$ $\begin{cases} ax + 2y + 2z = 2 \\ 2x + ay + 2z = a \\ 2x + 2y + az = 2 \end{cases}$

$|A| \neq 0$: én løsn.
$|A| = 0$: ingen løsn. eller uendelig mange løsn.

Vi løser $|A|=0$: $a^3 - 12a + 16 = 0$

$(a-2)^2(a+4) = 0$

$\underline{a=2}$ eller $a = \underline{-4}$

Alternativ

$a^3 - 12a + \underline{16} = 0$

$(a-2)\cdot(a^2+2a-8) = 0$

$\underline{a=2}$ eller $a^2+2a-8=0$

$\underline{a=2}$, $a=-4$

$(a^3-12a+16) : (a-2) = a^2+2a-8$

$-(a^3-2a^2)$

$2a^2-12a+16$

$-(2a^2-4a)$

$-8a+16$

$-8a+16$

0

Mulige heltallsløsn:

$a = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$\underline{a=2}$ er løsn.

$a=2$:

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{array}\right) \begin{array}{l} \\ \leftarrow -1 \\ \leftarrow -1 \end{array}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

y, z fri,
uendelig mange løsn.

$$2x + 2y + 2z = 2$$

$$\frac{2x}{2} = \frac{2 - 2y - 2z}{2}$$

$$\begin{cases} x = 1 - y - z \\ y = y \ \text{(fri)} \\ z = z \ \text{(fri)} \end{cases}$$

$$\underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1-y-z \\ y \\ z \end{pmatrix}, \quad y, z \text{ fri}$$

Konklusjon:

Uendelig mange løsn. for $a=2$, $a=-4$

Løsn. for $a=2$:

Løsn. for $a=-4$:

$a=-4$:

$$\left(\begin{array}{ccc|c} -4 & 2 & 2 & 2 \\ 2 & -4 & 2 & -4 \\ 2 & 2 & -4 & 2 \end{array}\right)$$

$x=0$
$y=1$
$z=0$
er løsn.

$\Downarrow$

uendelig mange løsn.

$$\left(\begin{array}{ccc|c} -4 & 2 & 2 & 2 \\ 2 & -4 & 2 & -4 \\ 2 & 2 & -4 & 2 \end{array}\right) \begin{array}{l} \\ \leftarrow \tfrac{1}{2} \\ \leftarrow \tfrac{1}{2} \end{array}$$

$$\downarrow$$

$$\left(\begin{array}{ccc|c} -4 & 2 & 2 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & 3 & -3 & 3 \end{array}\right) \begin{array}{l} \\ \\ \leftarrow 1 \end{array}$$

$$\left(\begin{array}{ccc|c} -4 & 2 & 2 & 2 \\ 0 & -3 & 3 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

z fri, uendelig mange fri

$$-4x + 2y + 2z = 2$$
$$-3y + 3z = -3$$

$$\frac{-3y}{-3} = \frac{-3 - 3z}{-3} \qquad y = 1 + z$$

$$-4x = 2 - 2y - 2z = 2 - 2(1+z) - 2z$$

$$\frac{-4x}{-4} = \frac{-4z}{-4} \qquad x = z$$

$$\left.\begin{array}{l} x = z \\ y = 1 + z \\ z = z \ \text{(fri)} \end{array}\right\} \quad \underline{x} = \begin{pmatrix} z \\ 1+z \\ z \end{pmatrix} \quad \begin{pmatrix} z \\ \text{fri} \end{pmatrix}$$

d)

$$A = \begin{pmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{pmatrix} \qquad \underline{b} = \begin{pmatrix} 2 \\ a \\ 2 \end{pmatrix} \qquad |A| = a^3 - 12a + 16 \neq 0$$

$$x = \frac{|A_1(\underline{b})|}{|A|} = \frac{0}{a^3-12a+16} = \underline{0} \qquad |A_1(b)| = \begin{vmatrix} 2 & 2 & 2 \\ a & a & 2 \\ 2 & 2 & a \end{vmatrix} = 0$$

(hvis to rader eller kolonner er like, så er det. $= 0$)

$$y = \frac{|A_2(b)|}{|A|} = \frac{a^3-12a+16}{a^3-12a+16} = \underline{1} \qquad |A_2(\underline{b})| = \begin{vmatrix} a & 2 & 2 \\ 2 & a & 2 \\ 2 & 2 & a \end{vmatrix} = |A|$$

$$z = \frac{|A_3(\underline{b})|}{|A|} = \frac{0}{a^3-12a+16} = \underline{0} \qquad |A_3(\underline{b})| = \begin{vmatrix} a & 2 & 2 \\ 2 & a & a \\ 2 & 2 & 2 \end{vmatrix} = 0$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}}$$ når $a^3-12a+16 \neq 0$ $(a \neq 2,\ a \neq -4)$

Husk:

1. $|A^T| = |A|$
2. Hvis to rader eller kolonner i A er like, så er $|A| = 0$.

② Funksjoner i flere (to) variable

Eks: $$f(x,y) = \frac{xy}{x+y}$$

Regne ut funksjonsverdier:

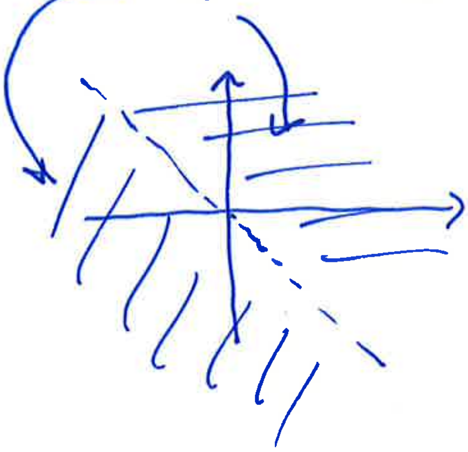
$$f(1,1) = \frac{1\cdot 1}{1+1} = \frac{1}{2}$$

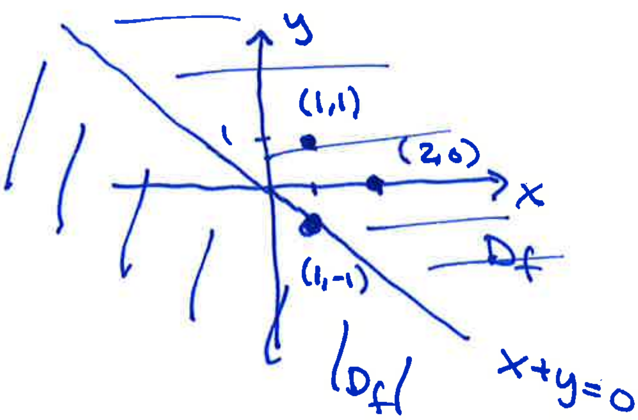
$$f(2,0) = \frac{2\cdot 0}{2+0} = 0$$

$$f(1,-1) = \frac{1\cdot(-1)}{1+(-1)}$$ ikke definert

Definisjonsområde:

D_f : $x+y \neq 0$





$$x+y=0$$
$$y=-x$$

Eks:
$f(x,y) = \frac{xy}{x+y}$

$f(1,1) = 1/2$
$\downarrow$
$(1,1,1/2)$ er med i grafen til f.

Grafen til f: $z = f(x,y)$

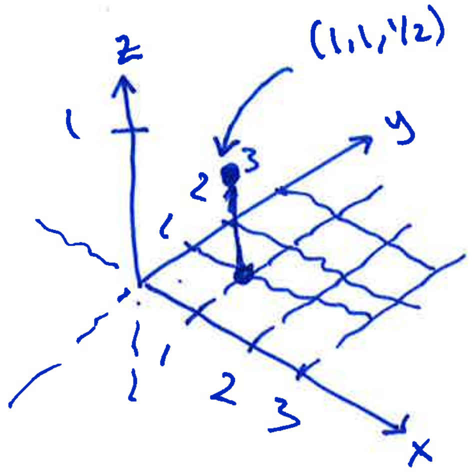
Grafen til f : Alle pkt. (x,y,z) slik at (x,y) er i D_f og $z = f(x,y)$

Kan tegne grafen til f i et 3d koordinatsystem.

Eks: $f(x,y) = \dfrac{xy}{x+y}$, $D_f: x+y \neq 0$

$f(1,1) = 1/2$

xyz-koordinatsystem:



Nivåkurver: $f(x,y) = c$ for et gitt tall c.

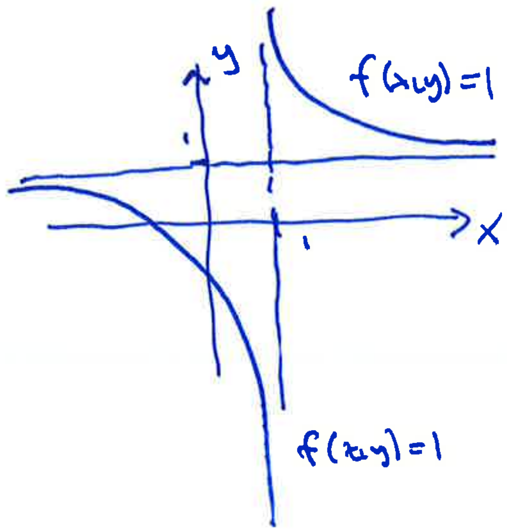
Eks: $f(x,y) = \boxed{\dfrac{xy}{x+y} = 1}$ ← alle pkt (x,y) i xy-planet der grafen til f har høyde $c=1$.

$$xy = x+y$$

$$xy - y = x$$

$$(x-1)y = x$$

$$y = \frac{x}{x-1} = \frac{x-1+1}{x-1} = 1 + \frac{1}{x-1}$$



Eks: $f(x,y) = x^2+y^2$

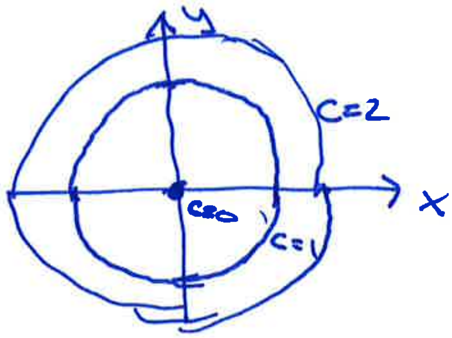
D_f: Alle punkt (x,y) $= \mathbb{R}^2$

Nivåkurver:

c=1: $x^2+y^2=1$

Sirkel med $r=1$

senter i $(0,0)$

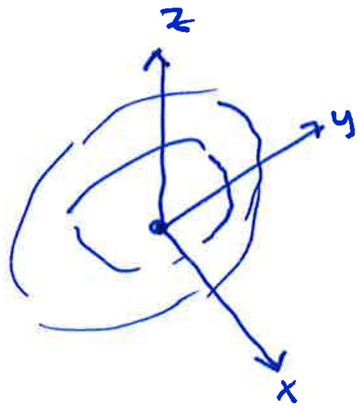


c=0: $x^2+y^2=0$

$(x,y)=(0,0)$

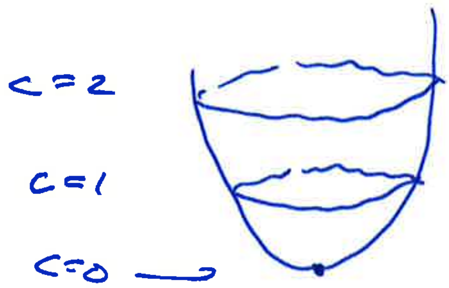
c=2: $x^2+y^2=2$

Sirkel $r=\sqrt{2} \simeq 1{,}41$



c=-1: $x^2+y^2=-1$

ingen punkt.



Eks:

$f(x,y) = c \qquad \dfrac{xy}{x+y} = c$

$xy = c(x+y)$

$xy - cy = cx$

$(x-c)y = cx$

$y = \dfrac{cx}{x-c} = c + \dfrac{c^2}{x-c}$

$cx : x - c = c$
$\underline{cx - c^2}$
$\quad c^2$

$\underline{c=0:}$ $xy = 0$
$x = 0$ eller $y = 0$

$\underline{c=1:}$ $y = 1 + \dfrac{1}{x-1}, \ x \neq 0$

$\underline{c=2:}$ $y = 2 + \dfrac{4}{x-2}, \ x \neq 0$