Plan

1 Om eksamen
2 Eksamen MET1180 06/2020

Husk: i) Kursevalueringen
ii) Forelesning fredag:
Padlet – skriv hva dere ønsker skal gjennomgås

Torsdag: Ekstra veiledning

1) Om eksamen:

Eksamenstid: 5t + 15min (10.00 – 15.15)

Omfang: 15 deloppgaver + bonusoppgaver
(15 × 6p = 90p) (kan hoppes over)

Krav til bestått: 40% – 60%

Oppgavetyper: Alle tidligere eksamensoppgaver er relevante.

Lurt å se på:
- tidligere eksamensoppgaver
- innleveringer (MET11806)

Tema:
- integrasjon
- matrise/vektorregning
- fn. i flere variabler
- fn i en variabel (derivasjon)

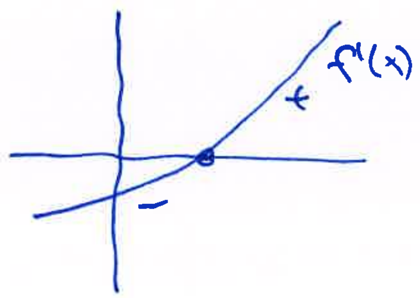
2 Eksamen MET1180 06/2020

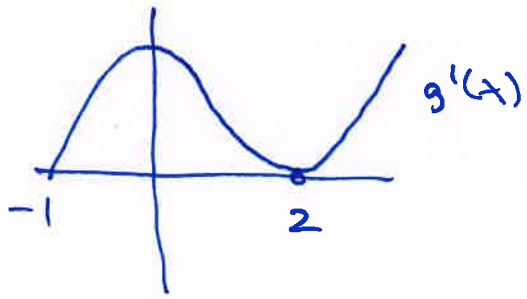
1. a) Stigningstallet til tangenten i $x=1$

i) $f'(1)=0$ ii) $g'(1) = \underline{\underline{2}}$

Vi leser av grafen til $f'(x)$ / $g'(x)$.

b) Minimumspunkt $\Rightarrow$ stasjonært punkt
eller
rand punkt





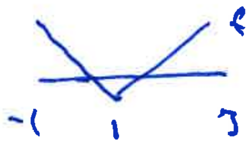
$\underline{\underline{x=1}}$

min. punkt

$\underline{\underline{x=-1}}$

min. punkt

c) i) f har ikke omvendt fn.
(ikke entydig invers siden f har et minimum i det indre)



ii) g har omvendt fn
(g er voksende i $[-1,3]$

2. $A = \begin{pmatrix} 3 & a & -2 \\ a & a^2+1 & -a \\ -2 & -a & 3 \end{pmatrix}$ $\quad \underline{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad \underline{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

a) $\underline{a=2}$: $\quad A \cdot \underline{x} = \underline{b}$

$$\left(\begin{array}{ccc|c} 3 & 2 & -2 & 1 \\ 2 & 5 & -2 & 0 \\ -2 & -2 & 3 & 1 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 2 & 5 & -2 & 0 \\ -2 & -2 & 3 & 1 \end{array}\right)$$

utvidet matrise

$$\to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 5 & -4 & -4 \\ 0 & -2 & 5 & 5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 6 & 6 \\ 0 & -2 & 5 & 5 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & 6 & 6 \\ 0 & 0 & 17 & 17 \end{array}\right)$$

trappeform

$$\begin{aligned} x \quad + z &= 2 & \quad x + 1 &= 2 \Rightarrow \underline{x=1} \\ y + 6z &= 6 & \quad y + 6 &= 6 \Rightarrow \underline{y=0} \\ 17z &= 17 & \quad \underline{z} &\underline{=1} \end{aligned}$$

En løsning: $\underline{\underline{(x,y,z) = (1,0,1)}}$

b) $|A| = \begin{vmatrix} 3 & a & -2 \\ a & a^2+1 & -a \\ -2 & -a & 3 \end{vmatrix} = 3\left(3(a^2+1) - a^2\right) - a \cdot (3a - 2a)$
$\qquad -2\left(-a^2 + 2(a^2+1)\right)$

$= 3(2a^2+3) - a^2 - 2(a^2+2) = \underline{\underline{3a^2+5}}$

$\boxed{A\underline{x} = \underline{b} \text{ har nøyaktig én løsning } \Leftrightarrow |A| \neq 0}$

$\underline{|A|=0}$: $3a^2+5=0$ ingen løsn. $(a^2 = -5/3)$ $\Rightarrow$ $A\underline{x}=\underline{b}$ har nøyaktig én løsning <u>for alle verdier av a</u>

c) Finn A^{-1} når $a=0$:

$a=0$: $A = \begin{pmatrix} 3 & 0 & -2 \\ 0 & 1 & 0 \\ -2 & 0 & 3 \end{pmatrix}$

$|A| = 3a^2+5 = 3\cdot 0^2+5 = \underline{5}$

$$A^{-1} = \frac{1}{|A|} \cdot \mathrm{adj}(A)$$

$$= \frac{1}{|A|} (C_{ij})^T$$

$$= \frac{1}{5} \begin{pmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{pmatrix}^T$$

$C_{11} = 3$ $C_{12} = 0$ $C_{13} = 2$

$C_{21} = 0$, .. 5 .. 0

.. 2 .. 0 .. 3

$$= \frac{1}{5} \begin{pmatrix} 3 & 0 & 2 \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{pmatrix}^T = \underline{\underline{\frac{1}{5} \begin{pmatrix} 3 & 0 & 2 \\ 0 & 5 & 0 \\ 2 & 0 & 3 \end{pmatrix}}}$$

d) $A^n \cdot \underline{b}$ $n \gg 0$, $a=2$

$A = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 5 & -2 \\ -2 & -2 & 3 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$$A \cdot \underline{b} = \begin{pmatrix} 3 & 2 & -2 \\ 2 & 5 & -2 \\ -2 & -2 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \underline{b}$$

$$A^2 \cdot \underline{b} = A(\underbrace{A \cdot \underline{b}}_{\underline{b}}) = A \cdot \underline{b} = \underline{b}$$

$$A^3 \cdot \underline{b} = A^2 \cdot (A \cdot \underline{b}) = A^2 \cdot \underline{b} = \underline{b}$$

$\vdots$

n stort heltall: $A^n \cdot \underline{b} = A^{n-1} \cdot (A\underline{b}) = A^{n-1} \cdot \underline{b}$

$$= A^{n-2} \cdot \underline{b} = \ldots = A\underline{b} = \underline{b} = \underline{\underline{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}}$$

3. a) Finn asymptotene til f:

$$f(x) = \frac{Q(x)}{L(x)} \begin{matrix} \leftarrow \text{grad } 2 \\ \leftarrow \text{grad } 1 \end{matrix} \qquad \frac{x^2}{x+1} = x-1+\frac{1}{x+1}$$

$$= \underset{\substack{\uparrow \\ \text{grad } 1}}{a(x)} + \frac{b \leftarrow \text{konst.}}{L(x)}$$

Asymptoter:

skrå: $y = a(x)$

vertikal: $L(x) = 0$

Leser av fra figur:

$y = -x-2$

$x = 2$

b) $f(x) = \underbrace{-x-2}_{\text{skrå}} + \frac{b}{\underbrace{x-2}_{\text{vertikale}}}$

Leser av: $x = 1,\ y = -2$

$$-2 = -1-2+\frac{b}{1-2}$$

$$-2 = -3+\frac{b}{-1}$$

$$1 = -b \quad \Rightarrow b = -1$$

$$f(x) = -x-2+\frac{-1}{x-2} = \frac{(-x-2)(x-2)-1}{x-2}$$

$$= \frac{-x^2+3}{x-2} = \frac{3-x^2}{x-2}$$

$$f'(x) = -1 - 1\cdot(-1)(x-2)^{-2}\cdot 1 = -1+\frac{1}{(x-2)^2}$$

c) Globalt maks/min $\Rightarrow$ stasjonært pkt

$$f'(x) = -1 + \frac{1}{(x-2)^2} = 0$$

$$\frac{1}{(x-2)^2} = 1$$

$$1 = (x-2)^2$$

$$x-2 = \pm\sqrt{1} = \pm 1$$

$$x = 2 \pm 1$$

$\underline{x=3}$ eller $\underline{x=1}$

Ser fra figur at ingen av disse er globalt maks/min.

$\Downarrow$

Ingen globale maks/min

4\. a) $$\int x(1-x)^2\,dx = \int x(1-2x+x^2)\,dx$$

$$= \int x - 2x^2 + x^3\,dx = \frac{1}{2}x^2 - \frac{2}{3}x^3 + \frac{1}{4}x^4 + C$$

potensregel

b) $$\int \frac{x}{1-x^2}\,dx = \int \frac{x}{u}\cdot\frac{du}{-2x} = -\frac{1}{2}\int\frac{1}{u}\,du$$

$\boxed{\begin{array}{l} u = 1-x^2 \\ du = -2x\cdot dx \end{array}}$ substitusjon

$$= -\frac{1}{2}\ln|u| + C = -\frac{1}{2}\ln|1-x^2| + C$$

c) $\displaystyle\int \frac{x}{(1-\sqrt{x})^2}\,dx = \int \frac{x}{u^2}(-2\sqrt{x})\,du$

$$\boxed{\begin{aligned} u &= 1-\sqrt{x} \\ du &= -\frac{1}{2\sqrt{x}}\,dx \end{aligned}} \qquad \sqrt{x} = 1-u$$

Substitusjon

$$= -2\int \frac{x\sqrt{x}}{u^2}\,du = -2\int \frac{(1-u)^3}{u^2}\,du$$

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

$$= -2\int \frac{1-3u+3u^2-u^3}{u^2}\,du$$

$$= -2\int u^{-2} - 3/u + 3 - u\,du$$

$$= -2\left(\frac{u^{-1}}{-1} - 3\ln|u| + 3u - \frac{1}{2}u^2\right) + C$$

$$= \frac{2}{1-\sqrt{x}} + 6\ln|1-\sqrt{x}| - 6(1-\sqrt{x}) + (1-\sqrt{x})^2 + C$$

d) $\displaystyle\int_{-4}^{a} f(x)\,dx = \underline{A(a)} \Longrightarrow A'(a) = f(a)$

Stasjonære pkt for A: $\underline{f(a) = 0}$ $=$ Skj. med x-aksen

i) Siden $[-4,3]$ er kompakt, så har A et globalt maks.

ii) Kandidater: $a = -3, -1, 2$ (stasj. pkt), $-4, 3$ (vendpkt.)

$A_3 > A_2$

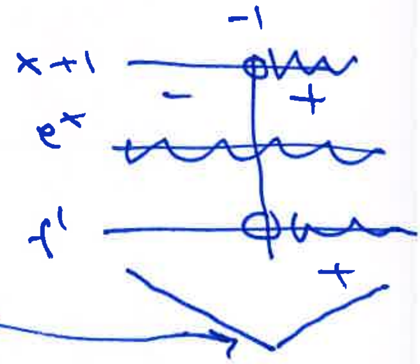
Ser fra figur at A (2) har størst verdi av disse

$\underline{\underline{a=2}}$

5. $f(x)=xe^x$

a) Forklar hvorfor $f(x)=-1$ ikke har løsn.

$f'(x)=1\cdot e^x + x\cdot e^x = (x+1)e^x$



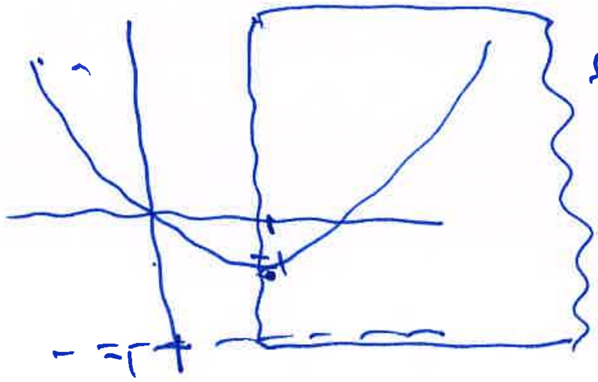
Globalt min:

$f(-1) = -1\cdot e^{-1} = -1/e$

$\simeq -0.37 > -1$

$\Downarrow$

$f(x)=-1$ har ingen løsn.



b) $f(x)=xe^x$, $x\geq -1$ f voksende

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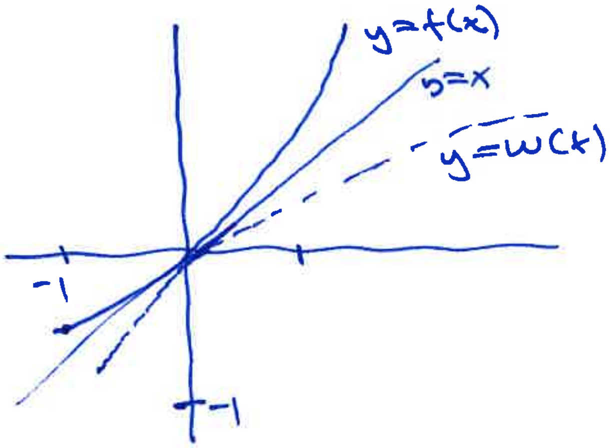
f har en omvendt fn. $W=f^{-1}$

Er W voksende / avtagende?



Vet fra teori: Grafen til $y=W(x)$ er speilbildet av grafen til $y=f(x)$ om linjen $y=x$

$W(f(x))=x$

$W'(f(x))\cdot f'(x)=1$

$W'(f(x)) = \frac{1}{f'(x)} > 0$ siden $f'(x)>0$

$\Rightarrow$ W er voksende

6. $C: \; 4x^2 - 24x + t^2y^2 = 64$ (t parameter)

a) $t \neq 0$: C ellipse

$$4x^2 - 24x + t^2y^2 = 64$$
$$4(x^2 - 6x + 9) + t^2(y^2) = 64 + 36$$
$$4(x-3)^2 + t^2 \cdot y^2 = 100$$

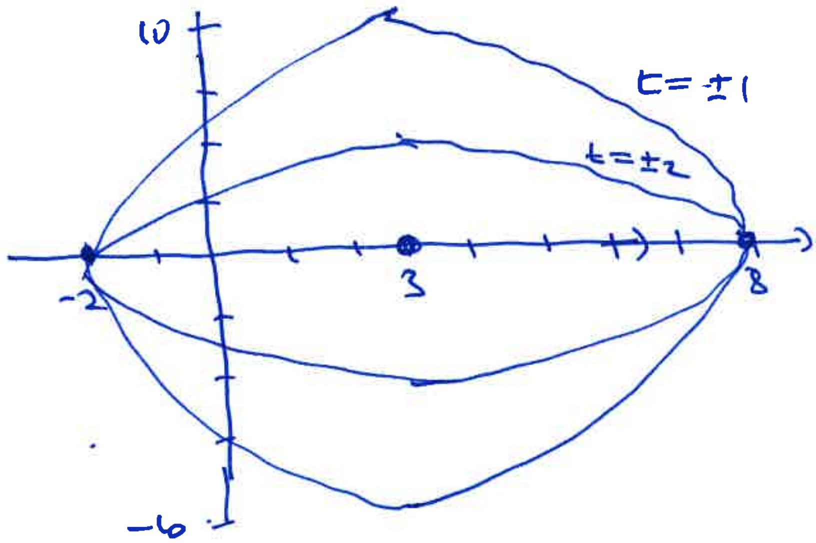
$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1 \qquad \frac{4(x-3)^2}{100} + \frac{t^2y^2}{100} = 1$$

$\left.\begin{array}{l} x_0 = 3 \\ y_0 = 0 \end{array}\right\}$ senter i $(3,0)$

$a^2 = 25$, $b^2 = 100/t^2$ — halvakser: $a = 5$, $b = 10/|t|$

$$\boxed{\begin{array}{l} C: \text{Ellipse med senter } (3,0) \\ \text{og halvakser } a=5, \; b = 10/|t| \\ \text{når } |t| \neq 0 \end{array}}$$

$t = \pm 1, \pm 2$



b) $f(x,y) = xy$

$f'_x = y = 0$

$f'_y = x = 0$

$\Downarrow$

Stasj. pkt: $(x,y) = (0,0)$

$H(f) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $\det = 0 - 1 = -1$

$\det < 0 \Rightarrow$ $(0,0)$ sadel-pkt

andre-derivert-testen

c) max $f(x,y) = xy$ når $\underbrace{4x^2 - 24x + 16y^2 = 64}_{C \text{ for } t=4}$

Lagrange problem:

$$L = xy - \lambda(4x^2 - 24x + 16y^2 - 64)$$

$$\boxed{\begin{aligned} L'_x &= y - \lambda(8x - 24) = 0 \\ L'_y &= x - \lambda(32y) = 0 \\ 4x^2 - 24x + 16y^2 &= 64 \end{aligned}}$$

← løser Lagrange-betingelsene

$\Downarrow$

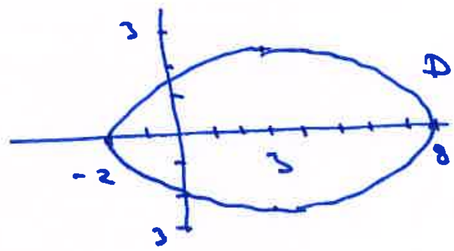
kandidater for maks

D = mengden av tillatte pkt

= C for $t = 4$

(fra a) = ellipse, sentr $(3,0)$

halvakser $a = 5$, $b = 10/4 = 5/2$



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i) D er kompakt $\overset{\text{EVS}}{\Longrightarrow}$ Vi har et maks

ii) ingen tillatte pkt med degenerert bibetingelse

(1) $\lambda = \dfrac{y}{8x-24}$

(2) $\lambda = \dfrac{x}{32y}$

$$\frac{y}{8x-24} = \frac{x}{32y}$$

$$32y^2 = x(8x-24)$$

$$32y^2 = 8x^2 - 24x \quad | : 2$$

$$16y^2 = 4x^2 - 12x$$

$$y^2 = \frac{4x^2-12x}{16} = \frac{x^2}{4} - \frac{3x}{4}$$

(3) $4x^2 - 24x + \underline{16y^2} = 64$

$$4x^2 - 24x + (4x^2 - 12x) = 64$$

$$8x^2 - 36x - 64 = 0 \quad :4$$

$$2x^2 - 9x - 16 = 0$$

$$x = \frac{9 \pm \sqrt{81 - 4\cdot 2\cdot(-16)}}{4} = \frac{9 \pm \sqrt{209}}{4}$$

$x_1 \simeq \underline{5.864}$ $\quad x_2 \simeq \underline{-1.364}$

$y_1 \simeq \underline{\pm 2.049}$ $\quad y_2 \simeq \underline{\pm 1.220}$

Maks: $(x, y) = \underline{\underline{(5.864,\ 2.049)}}$