Plan

① Delvis integrasjon

Metode for å integrere et produkt.

Formel:

$$\boxed{\int u' \cdot v\,dx = u \cdot v - \int u v'\,dx}$$

Ex: $\int x \cdot \ln x\,dx =$

$$\boxed{\begin{array}{ll} u = \frac{1}{2}x^2 & v = \ln x \\ u' = x & v' = 1/x \end{array}}$$

$$= \frac{1}{2}x^2 \cdot \ln x - \int \frac{1}{2}x^2 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{2}x^2 \cdot \ln x - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\left(\frac{1}{2}x^2\right) + C$$

$$= \underline{\underline{\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C}}$$

Ex: $\int x \cdot e^x\,dx =$

$$\boxed{\begin{array}{ll} u = \cancel{\tfrac{1}{2}x^2}\ e^x & v = \cancel{e^x}\ x \\ u' = \cancel{x}\ e^x & v' = \cancel{e^x}\ 1 \end{array}}$$

$$e^x \cdot x - \int e^x \cdot 1\,dx$$

$$= x \cdot e^x - \int e^x\,dx = \underline{\underline{xe^x - e^x + C}}$$

Eks: $\int \ln(x)\,dx = \int 1\cdot\ln(x)\,dx$

$$u = x \qquad v = \ln x$$
$$u' = 1 \qquad v' = 1/x$$

$$= x\cdot\ln x - \int x\cdot\frac{1}{x}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= \underline{x\ln x - x + C}$$

Sjekk:

$$(x\cdot\ln x - x + C)' =$$

$$1\cdot\ln x + \cancel{x\cdot\frac{1}{x}}\ \cancel{-1} + 0 = \ln x$$

Viktig formel:

$$\boxed{\int \ln x\,dx = x\ln x - x + C}$$

(2) Substitusjon

Kjerneregelen:

ytre funksjon

kjernen, indre funksjon

Eks: $f(x) = e^{2x} = e^u, \quad u = 2x$

$$f'(x) = (e^u)'_u \cdot u'_x$$
$$= e^u\cdot 2 = 2e^u = \underline{2e^{2x}}$$

Formel for substitusjon:

$$du = u'\,dx$$

Framgangsmåte:

- velg uttrykket u
- bruk uttrykket for u og formelen $du = u'dx$ til å skrive om integralet til et integral i u: $\int g(u)\,du$
- løs integralet i u

Eks: Substitusjon

$$\int e^{2x}\,dx = \int e^u\,dx$$

$$u = 2x$$
$$du = u'\cdot dx = 2\,dx$$
$$dx = \frac{1}{2}\,du$$

$$= \int e^u\cdot\frac{1}{2}\,du = \frac{1}{2}\int e^u\,du$$

$$= \frac{1}{2}e^u + C = \underline{\frac{1}{2}e^{2x} + C}$$

Eks:

$$\int (1-x)^4\,dx = \int u^4\,dx = \int u^4 \cdot \frac{1}{-1}\,du \leftarrow u'$$

$$\boxed{\begin{array}{l} u = 1-x \\ du = -1 \cdot dx \end{array}} \quad dx = \frac{1}{-1}\cdot du = -1\cdot du$$

$$= -\int u^4\,du = -\left(\frac{1}{5}u^5\right) + C = \underline{\underline{-\frac{1}{5}(1-x)^5 + C}}$$

Eks:

$$\int \frac{2x}{1-x^2}\,dx = \int \frac{\cancel{2x}}{u} \cdot \frac{1}{(-\cancel{2x})}\,du$$

$$\boxed{\begin{array}{l} u = 1-x^2 \\ du = -2x\,dx \end{array}} \rightarrow dx = \frac{1}{-2x}\,du$$

$$= \int -\frac{1}{u}\,du = -\int \frac{1}{u}\,du = -\ln|u| + C$$

$$= \underline{\underline{-\ln|1-x^2| + C}}$$

Eks:

$$\int x e^{-x^2}\,dx = \int \cancel{x} e^u \cdot \frac{1}{-2\cancel{x}}\,du$$

$$\boxed{\begin{array}{l} u = -x^2 \\ du = -2x\,dx \end{array}} \rightarrow dx = \frac{1}{-2x}\,du$$

$$= \int e^u \cdot \frac{1}{-2}\,du = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C$$

$$= \underline{\underline{-\frac{1}{2}e^{-x^2} + C}}$$

Ex:
$$\int \frac{e^{1-\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^{1-u}}{\cancel{u}} \cdot 2\cancel{\sqrt{x}}\,du$$

$$\boxed{\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \end{aligned}} \rightarrow dx = 2\sqrt{x}\,du$$

$$= \int e^{1-u} \cdot 2\,du = 2\int e^{1-u}\,du = 2\int e^{v}\,\frac{1}{-1}\,dv$$

$$\boxed{\begin{aligned} v &= 1-u \\ dv &= -1 \cdot du \end{aligned}}$$

$$= -2\int e^{v}\,dv = -2e^{v} + C = -2e^{1-u} + C$$

$$= \underline{\underline{-2e^{1-\sqrt{x}} + C}}$$

Alt:
$$\int \frac{e^{1-\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{e^{u}}{\cancel{\sqrt{x}}} \cdot (-2\cancel{\sqrt{x}}\,du)$$

$$\boxed{\begin{aligned} u &= 1-\sqrt{x} \\ du &= -\frac{1}{2\sqrt{x}}\,dx \end{aligned}} \rightarrow dx = -2\sqrt{x}\,du$$

$$= \int e^{u} \cdot (-2)\,du = -2\int e^{u}\,du = -2e^{u} + C$$

$$= \underline{\underline{-2e^{1-\sqrt{x}} + C}}$$