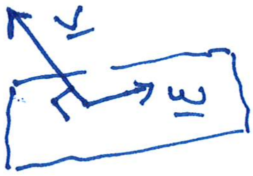
	Plan
1	Repetisjon og oppgaveregning
2	Tangenten til en nivåkurve
3	Gradienten og den retningsderiverte

Gradient/retn. derivert utsatt til torsdag

① Oppgaveark 39

2d) $\underline{v} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix}$ $\quad \underline{w} \perp \underline{v} \iff \underline{v} \cdot \underline{w} = \begin{pmatrix} 4 \\ 7 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0$



$$4x + 7y - 3z = 0$$

$$\frac{4x}{4} = \frac{-7y + 3z}{4}$$

$$x = -\frac{7}{4}y + \frac{3}{4}z$$

$(y, z \text{ fri})$

$$\underline{w} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -7y/4 + 3z/4 \\ y \\ z \end{pmatrix} = \frac{y}{4} \cdot \begin{pmatrix} -7 \\ 4 \\ 0 \end{pmatrix} + \frac{z}{4} \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$$

= alle lin. komb. av $\begin{pmatrix} -7 \\ 4 \\ 0 \end{pmatrix}$ og $\begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$

4 a) $f(x,y) = 17x^{1.2}y^{3.4} = 17x\sqrt[5]{x}\; y^3 \cdot \sqrt[5]{y^2}$

$D_f = \{(x,y) : x, y \geq 0\}$

$V_f = [0, \rightarrow)$

$$f(-1,1) = 17 \cdot (-1) \cdot \sqrt[5]{-1} \cdot 1$$

$$\sqrt[5]{-1} = (-1)^{1/5} = (-1)^{2/10}$$

$$1 = \sqrt[10]{(-1)^2} \qquad = (-1)^{2/10}$$

$$= \text{ikke def. } \left(\sqrt[10]{-1}\right)^2$$

10 g) $f(x,y) = x^2y^2 - x^2 - y^2 + 3$

$f'_x = \underline{2xy^2 - 2x} = 2x(y^2-1) = 0$ $\quad$ $\underline{x=0}$ eller $\underline{y=\pm 1}$

$f'_y = \underline{x^2 \cdot 2y - 2y} = 2y(x^2-1) = 0$ $\quad$ $\underline{y=0}$ " $\underline{x=\pm 1}$

$\underline{\text{Stasj. pkt}}$: $(x,y) = (0,0), (\pm 1, \pm 1)$ $\quad$ $x=0, y=0$

$= (0,0), (1,1), (-1,1),$ $\quad$ ~~$x=0, x=\pm 1$~~ $\quad$ ~~$y=\pm 1, y=0$~~

$(1,-1), (-1,-1)$ $\quad$ $y=\pm 1, x=\pm 1$

$$H(f) = \begin{pmatrix} 2y^2-2 & 4xy \\ 4xy & 2x^2-2 \end{pmatrix}$$

Andredrivert-teste

$\underline{(0,0)}$: $H(f)(0,0) = \begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ $\quad$ $\det = 4 > 0$, $\text{tr} = -4 < 0$ $\Rightarrow$ $(0,0)$ $\underline{\text{lokalt max}}$

$(1,1)$: $H(f)(1,1) = \begin{pmatrix} 0 & 4 \\ 4 & 0 \end{pmatrix}$ $\quad$ $\det = -16 < 0$ $\Downarrow$ $\underline{\text{sadelpkt}}$

$(-1,-1)$: $H(f)(-1,-1)$ $//$

$(1,-1)$: $H(f)(1,-1) = \begin{pmatrix} 0 & -4 \\ -4 & 0 \end{pmatrix}$ $\quad$ $\det = -16 < 0$ $\Downarrow$ $\underline{\text{sadelpkt}}$

$(-1,1)$: $H(f)(-1,1)$ $//$

① $\underline{\text{Stasjonære pkt}}$

$f'_x = f'_y = 0$

② $\underline{\text{Andredrivert-teste}}$

$\det > 0$, $\text{tr} > 0$ $\}$ lokalt min

$\det > 0$, $\text{tr} < 0$ $\}$ lokalt maks

$\det < 0$ sadelpkt

10h) $f(x,y) = \sqrt{x^2+y^2} = \sqrt{u} = u^{1/2}$, $u = \underline{x^2+y^2}$

$f'_x = \frac{1}{2}u^{-1/2} \cdot 2x = \frac{\cancel{2}x}{\cancel{2}\sqrt{u}} = \frac{x}{\sqrt{x^2+y^2}} = 0 \Rightarrow x = 0$

$f'_y = \frac{1}{2}u^{-1/2} \cdot 2y = \frac{\cancel{2}y}{\cancel{2}\sqrt{u}} = \frac{y}{\sqrt{x^2+y^2}} = 0 \Rightarrow y = 0$

$\Rightarrow$ kandidat $(x,y) = (0,0)$, men $f'_x(0,0)$ og $f'_y(0,0)$ er <u>ikke definert</u>

<u>Ingen stasjonære pkt</u>

$$H(f) = \begin{pmatrix} \frac{y^2}{u\sqrt{u}} & -\frac{xy}{u\sqrt{u}} \\ -\frac{xy}{u\sqrt{u}} & \frac{x^2}{u\sqrt{u}} \end{pmatrix}$$

<u>Kritisk pkt:</u>
Et pkt (x,y) der f'_x eller f'_y <u>ikke</u> er definert

$$f''_{xx} = \left(\frac{x}{\sqrt{u}}\right)'_x = \left(x \cdot u^{-1/2}\right)'_x = 1 \cdot u^{-1/2} + x \cdot (-1/2)\, u^{-3/2} \cdot 2x$$

$$= \frac{u \cdot 1}{u \cdot \sqrt{u}} - \frac{\cancel{2}x^2}{\cancel{2}u\sqrt{u}} = \frac{\overset{x^2+y^2}{u} - x^2}{u\sqrt{u}} = \frac{y^2}{(x^2+y^2)\sqrt{x^2+y^2}}$$

$$f''_{xy} = \left(x \cdot u^{-1/2}\right)'_y = x \cdot (-\tfrac{1}{\cancel{2}})\, u^{-3/2} \cdot \cancel{2}y = -\frac{xy}{u\sqrt{u}}$$

$$f''_{yy} = \left(y \cdot u^{-1/2}\right)'_y = \frac{x^2}{u\sqrt{u}}$$

11. a) $f(x,y) = xy(x^2-y^2) = x^3y - xy^3$

$f'_x = 3x^2y - y^3 = y(3x^2-y^2) = 0$ $\quad$ $y=0$ eller $3x^2=y^2$

$f'_y = x^3 - 3xy^2 = x(x^2-3y^2) = 0$ $\quad$ $x=0$ " $x^2=3y^2$

$y=0,\ x=0$

~~$y=0,\ x^2=3y^2$~~

~~$x=0,\ 3x^2=y^2$~~

$3x^2=y^2,\ x^2=3y^2$

$x^2=3\cdot(3x^2)$

$-8x^2=0$

$x=0$

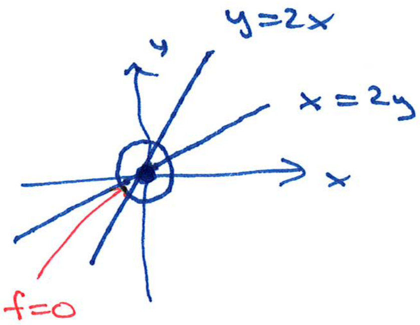
$y=0$

Stasjonære pkt:

$(x,y) = (0,0)$

$$H(f) = \begin{pmatrix} 6xy & 3x^2-3y^2 \\ 3x^2-3y^2 & -6xy \end{pmatrix}$$

$(0,0)$: $\quad H(f)(0,0) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad$ det $=0$ ingen konkl. fra andrederivert-testen



Bruker defn:

$f(0,0) = 0$

$f(x,y) = f(2y,y)$

$= 2y\cdot y\cdot((2y)^2 - y^2)$

$= 2y^2\cdot 3y^2 = 6y^4 > 0$

$f(x,2x) = x\cdot 2x\,(x^2-(2x)^2)$

$= 2x^2(-3x^2) = -6x^4 < 0$

$\Rightarrow$ $(0,0)$ er sadelpkt

5c) $f(x,y)=c$ nivåkurve i høyde $z=c$

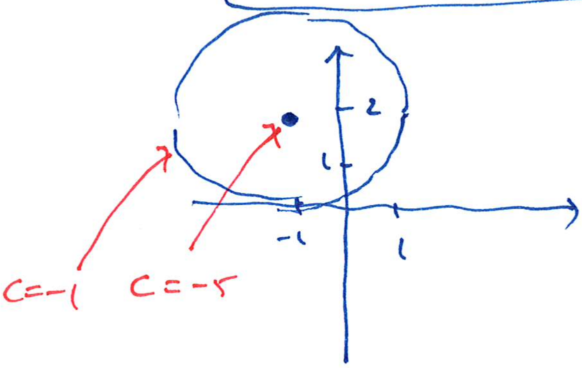
$$x^2+2x+y^2-4y=c$$

$$\underbrace{x^2+2x+1}_{(x+1)^2}\ \underbrace{+y^2-4y+4}_{(y-2)^2} = c+1+4$$

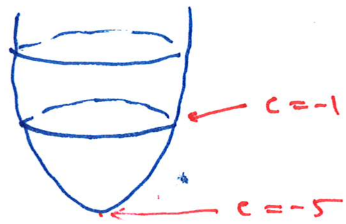
$$(x-x_0)^2+(y-y_0)^2=r^2$$

$$(x+1)^2+(y-2)^2=c+5$$

Sirkel m/sentrum $(-1,2)$, $r=\sqrt{c+5}$ hvis	$c>-5$
pkt $(-1,2)$	$c=-5$
ingen pkt.	$c<-5$



6c



$f_{min} = \underline{\underline{-5}}$ i $(x,y)=(-1,2)$

(2) Tangenten til en nivåkurve

$$\frac{(x-x_0)^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

Eks: (5c) $f(x,y) = x^2 - 2x + 4y^2$

Nivåkurve: $f(x,y) = c$

$$x^2 - 2x + 1 + 4y^2 = c + 1$$

$$(x-1)^2 + 4\cdot y^2 = c+1 \quad | : (c+1)$$

$$\frac{(x-1)^2}{c+1} + \frac{\cancel{4}y^2 \cdot \cancel{1/4}}{c+1 \cdot 1/4} = 1$$

$$\frac{(x-1)^2}{c+1} + \frac{y^2}{(c+1)/4} = 1$$

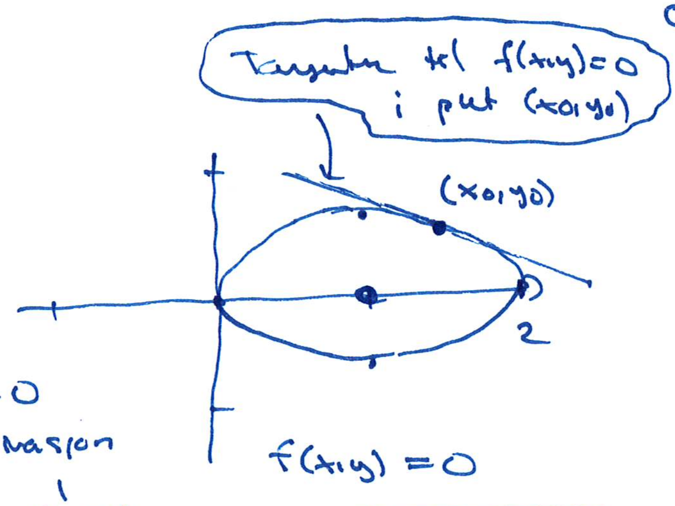
Ellipse m/ sentrum $(1,0)$, halvakser $a = \sqrt{c+1}$ og $b = \frac{\sqrt{c+1}}{\sqrt{4}} = \frac{\sqrt{c+1}}{2}$ for $c > -1$

Pkt $(1,0)$ $c = -1$

Ingen pkt $c < -1$

Eks: $c = 0$

$a = 1$ $b = 1/2$



Finner y': $f(x,y) = 0$

$$x^2 - 2x + 4y^2 = 0$$

{ implisitt derivasjon

$$\underbrace{2x - 2}_{f'_x} + \underbrace{8y}_{f'_y} \cdot y' = 0$$

$$y' = \frac{-2x+2}{8y} = -\frac{x-1}{4y} \qquad \frac{8y \cdot y'}{8y} = \frac{-2x+2}{8y}$$

$$\boxed{\textbf{Generelt:} \quad f(x,y) = c \;\Downarrow\; y' = -\frac{f'_x}{f'_y}}$$

<u>Merk:</u> Hvis $f(x,y)$ er gitt og definert for alle (x,y), så <u>ligger ethvert punkt (x_0,y_0) på en nivåkurve</u>.

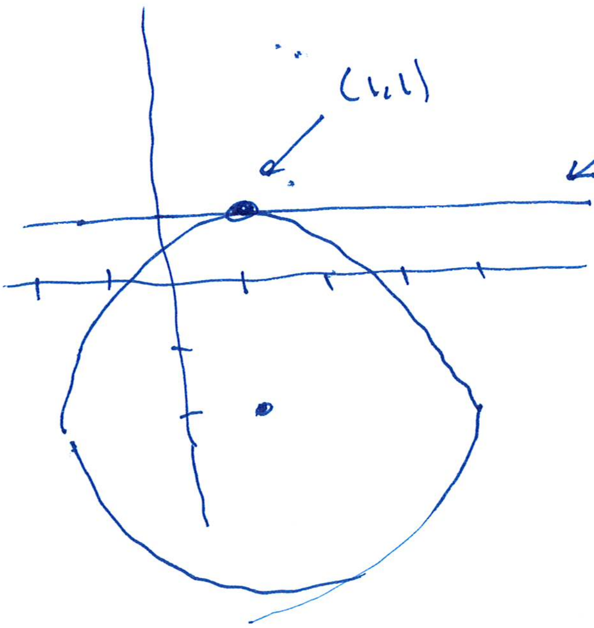
$$\boxed{f(x_0,y_0)=c \Rightarrow (x_0,y_0) \text{ ligger på } f(x,y)=c}$$

<u>Eks:</u> $f(x,y) = x^2-2x+y^2+4y$

$(1,1)$: $f(1,1) = 1-2+1+4 = \underline{4} \Rightarrow (1,1)$ ligger på $\underline{f(x,y)=4}$.

<u>Nivåkurve:</u> $f(x,y)=4$

$$x^2-2x+1+y^2+4y+4 = 4+1+4$$
$$(x-1)^2+(y+2)^2 = 9 \quad \leftarrow \text{ sirkel m/ senter } \underline{(1,-2)} \text{ og } r=\sqrt{9}=\underline{3}$$



$$y' = -\frac{f'_x}{f'_y} = -\frac{2x-2}{2y+4}$$

$$y'(1,1) = -\frac{0}{6} = \underline{0}$$

<u>Tangent:</u>

$$y-1 = 0\cdot(x-1)$$
$$y-1=0$$
$$\underline{\underline{y=1}}$$