	Plan	
1	Oppgavesett 43: Oppgave 2op, 4d, 5d, 6	
2	Optimering med bibetingelser	

Kontor: B3y-085
(fra 27/03)

① Oppgavesett 43

2o $x^2y^2 - x^2 - y^2 = 0$
$+1$

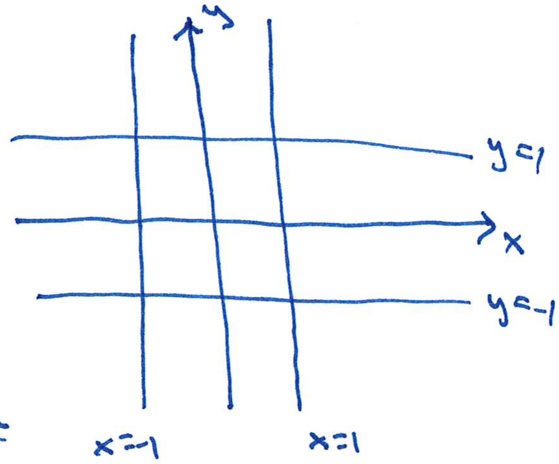
lukket: ok
begrenset: nei

$x^2y^2 - x^2 - y^2 + 1 = 0$

$(x^2 - 1)(y^2 - 1) = 0$

$x^2 - 1 = 0$ eller $y^2 - 1 = 0$

$x = \pm 1$ " $y = \pm 1$



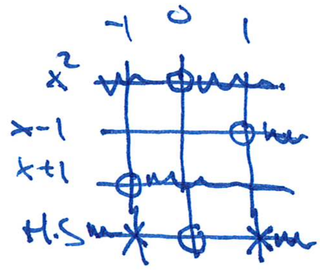
p) $x^2y^2 - x^2 - y^2 + 1 = 1$

lukket: ok
begrenset: nei

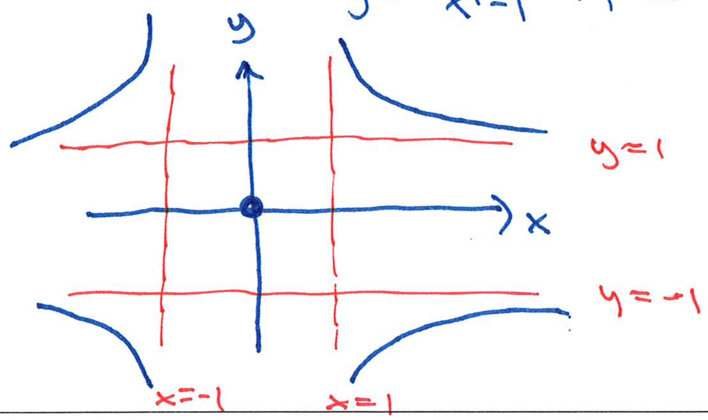
$(x^2-1)(y^2-1) = 1$

$y^2 - 1 = \frac{1}{x^2-1}$

$y^2 = \frac{1}{x^2-1} + 1 = \frac{\cancel{1} + x^2 - \cancel{1}}{x^2-1} = \frac{x^2}{x^2-1}$



Vi har punkter med $x > 1$, $x < -1$ og $x = 0$



4 d) max : $(-4,-4), (4,4)$

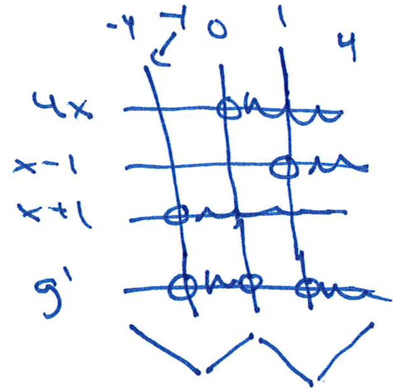
min : $(0,0)$?

$$g(x,y) = x^2y^2 - x^2 - y^2 + 1$$

$\rightarrow \underline{y=x}$: $x^4 - 2x^2 + 1 = g(x)$

$4x^3 - 4x = g'(x)$

$g'(x) = 4x(x^2-1)$



$\underline{\text{min}}$: $\left.\begin{array}{l} g(1,1) = 0 \\ g(-1,-1) = 0 \end{array}\right\}$ min

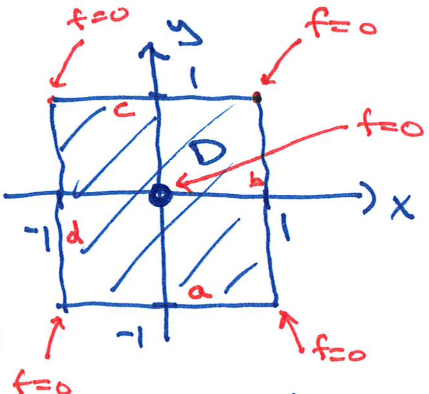
$\underline{\text{max}}$: $\left.\begin{array}{l} g(4,4) = 256 - 16 - 16 + 1 = 225 \\ g(-4,-4) = \text{—} \; '' \; \text{—} = 225 \end{array}\right\}$ max

$g(0,0) = 1$

5 a) max/min $f(x,y) = xy(x^2-y^2)$

$= x^3y - xy^3$

når $-1 \le x,y \le 1$



lukket og begrenset

$\Rightarrow$ det fins max/min

$\Rightarrow$ max/min = kandidatpkt med størst/minst $\underline{f\text{-verdi}}$

i) $\underline{\text{Stasjonære pkt}}$

$\boxed{f(0,0) = 0}$

$f'_x = 3x^2y - y^3 = 0$

$f'_y = x^3 - x \cdot 3y^2 = 0$

$\underline{y}\,(3x^2 - y^2) = 0$

$\underline{x}\,(x^2 - 3y^2) = 0$

i) $y=0, x=0$: $\underline{(0,0)}$

ii) $y=0, x^2-3y^2=0$: $(0,0)$

iii) $3x^2-y^2=0, x=0$: $(0,0)$

iv) $3x^2-y^2=0, x^2-3y^2=0$

$y^2 = 3x^2 \Rightarrow x^2 - 3\cdot 3x^2 = 0$

$-8x^2 = 0 \Rightarrow x=0 \Rightarrow y=0$

$(0,0)$

Randpkt: Kanter a, b, c, d $\quad f(x,y) = xy(x^2-y^2)$

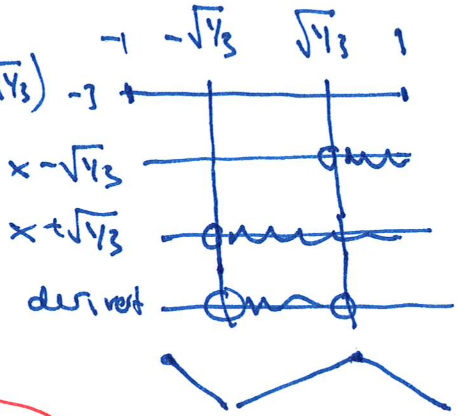
a: $y=-1$, $-1 \le x \le 1$

$f(x,-1) = -x^3 + x$

$(-x^3+x)' = -3x^2+1 = 0 \quad = -3(x-\sqrt{1/3})(x+\sqrt{1/3})$

$x^2 = 1/3$

$x = \pm\sqrt{1/3}$



$$\boxed{\frac{2}{3}\cdot\sqrt{\frac{1}{3}} = \frac{2\cdot\sqrt{1}}{3\cdot\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{9}}$$

max på a: $f(-1,-1) = 0$

$f(\sqrt{1/3},-1) = -\sqrt{1/3}\left(\frac{1}{3}-1\right) = \frac{2}{3}\sqrt{1/3}$ ← max på a)

min på a: $f(-\sqrt{1/3},-1) = \sqrt{1/3}\cdot\left(\frac{1}{3}-1\right) = -\frac{2}{3}\sqrt{1/3}$ ← min på a)

$f(1,-1) = 0$

b) $x=1$, $-1 \le y \le 1$

$f(1,y) = y - y^3$

symmetri, bruker a)

max på b): $f(1,\sqrt{1/3}) = \frac{2}{3}\sqrt{\frac{1}{3}}$

min –"–: $f(1,-\sqrt{1/3}) = -\frac{2}{3}\sqrt{1/3}$

c) $y=1$, $-1 \le x \le 1$

$f(x,1) = x^3 - x$

symmetri, bruker a) $\cdot(-1)$

max på c: $f(-\sqrt{1/3},1) = \frac{2}{3}\sqrt{1/3}$

min –"–: $f(\sqrt{1/3},1) = -\frac{2}{3}\sqrt{1/3}$

d): $x=-1$, $-1 \le y \le 1$

$f(-1,y) = -y + y^3$

symmetri, bruker c:

max på d: $f(-1,-\sqrt{1/3}) = \frac{2}{3}\sqrt{1/3}$

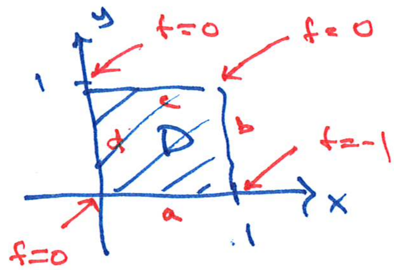
min –"– $f(-1,\sqrt{1/3}) = -\frac{2}{3}\sqrt{1/3}$

Konkl: $f_{max} = \underline{\underline{\frac{2}{3}\sqrt{\frac{1}{3}}}}$ for $(x,y) = (\sqrt{1/3},-1), (1,\sqrt{1/3}), (-\sqrt{1/3},1), (-1,-\sqrt{1/3})$

$f_{min} = -\underline{\underline{\frac{2}{3}\sqrt{\frac{1}{3}}}}$ for $(x,y) = (-\sqrt{\frac{1}{3}},-1), (1,-\sqrt{\frac{1}{3}}), (\sqrt{\frac{1}{3}},1), (-1,\sqrt{\frac{1}{3}})$

<u>6</u>. max/min $f(x,y) = \sqrt{xy} - x$

når $0 \le x,y \le 1$



$$f(x,y) = \sqrt{x}\cdot\sqrt{y} - x = x^{1/2}\cdot y^{1/2} - x$$

$$f'_x = \frac{1}{2\sqrt{x}}\cdot\sqrt{y} - 1 = \frac{\sqrt{y}}{2\sqrt{x}} - 1$$

$$f'_y = \sqrt{x}\cdot\frac{1}{2\sqrt{y}} = \frac{\sqrt{x}}{2\sqrt{y}}$$

Ekstremverdisetn: det fins max/min

i) <u>Indre stasjonære pkt:</u>

$f'_x = f'_y = 0$: $\frac{\sqrt{x}}{2\sqrt{y}} = 0 \Rightarrow \sqrt{x} = 0 \Rightarrow x = 0$ <u>umulig</u>

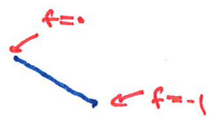
ii) <u>Indre kritiske pkt</u>

Indre pkt: $x > 0,\ y > 0 \Rightarrow$ ingen andre kritiske pkt.

iii) <u>Randpkt:</u>

<u>a</u>: $y = 0$ $f(x,0) = -x$

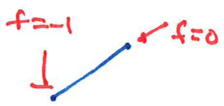
$(-x)' = -1$



max: $f = 0$
min: $f = -1$

<u>b</u>: $x = 1$ $f(1,y) = \sqrt{y} - 1$

$(\sqrt{y} - 1)' = \frac{1}{2\sqrt{y}} > 0$



max: $f = 0$
min: $f = -1$

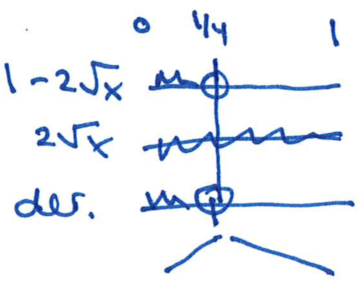
<u>c</u>: $y = 1$ $f(x,1) = \sqrt{x} - x$

$$(\sqrt{x} - x)' = \frac{1}{2\sqrt{x}} - 1 = \frac{1 - 2\sqrt{x}}{2\sqrt{x}} = 0$$

$1 - 2\sqrt{x} = 0$

$\sqrt{x} = 1/2$

$x = 1/4$



max: $f(1/4, 1) = \frac{1}{4}$
min $f = 0$

d: $x=0$ $f(0,y)=0$ max/min: $f=0$

Konklusjon:

$f_{max} = 1/4$ i $(x,y) = (1/4, 1)$

$f_{min} = -1$ i $(x,y) = (1,0)$